Partial Differential Equations in Analysis and Mathematical Physics

Santa Margherita di Pula, Cagliari. 30 May – 1 June, 2019

Partial Differential Equations in Analysis and Mathematical Physics

May 30 - June 1, 2019
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http://sites.unica.it/pdeamp/

Plenary speakers:
José Antonio Carrillo de la Plata (Imperial College of London, UK)
Gregorio Falqui (Università di Milano Bicocca, Italy)
Sara Lombardo (Loughborough University, UK)
Filomena Pacella (Università di Roma La Sapienza, Italy)
Susanna Terracini (Università di Torino, Italy)
Vincenzo Vespri (Università di Firenze, Italy)

Organizing committee:
Claudia Anedda, Lucio Cadeddu, Fabrizio Cuccu, Francesca Damagotis, Antonio Greco, Antonio Iannizzotto, Monica Marras, Cornelis Van der Mee, Giuseppe Viglizzo

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1 INTRODUCTION AND ACKNOWLEDGMENTS

We are delighted to be hosting the Conference Partial Differential Equations in Analysis and Mathematical Physics (PDEAMP). The aim of this activity is to gather international researchers, from the areas of Mathematical Physics and Analysis, specialists in related subjects, and let them share their results and discuss further developments and open problems.

The PDEAMP consists of 6 plenary lectures and 3 special sessions (SS).

SS1: Local and Non-Local Elliptic Equations and Applications. Organizers

Prof. Filomena Pacella (Università la Sapienza di Roma, Italy)
Prof. Enrico Valdinoci (University of Western Australia, Australia)

SS2: Aggregation–Diffusion PDEs and Applications. Organizers

Prof. José Antonio Carrillo de la Plata (Imperial College of London, UK)
Prof. Vincenzo Vespri (Università di Firenze, Italy)

SS3: Nonlinear Evolution Equations: Analytical and Geometrical Methods and New Perspectives. Organizers

Prof. Gregorio Falqui (Università di Milano Bicocca, Italy)
Prof. Sara Lombardo (Loughborough University, UK)

Mainly, this meeting was possible thanks to the generous availability of all the speakers who participated in the activity. Moreover, we also appreciate the effort of all the other colleagues who, directly or indirectly, offered their useful contribution in the organization of the PDEAMP. Finally, we would like to express our gratitude to Fondazione di Sardegna, the Dipartimento di Matematica ed Informatica (Università di Cagliari) and the INdAM–GNAMPA: Istituto Nazionale di Alta Matematica “Francesco Severi”–Gruppo Nazionale per l’Analisi Matematica, la Probabilità e le loro Applicazioni; without the support of these sponsors the conference would have been not possible.

Cagliari, 30 May 2019

The Scientific Committee

Claudia Anedda, Lucio Cadeddu, Fabrizio Cuccu, Francesco Demontis, Antonio Greco, Antonio Iannizzotto, Monica Marras, Cornelis Van der Mee and Giuseppe Viglialoro, of the University of Cagliari.
Nonlinear Aggregation-Diffusion Equations in the Diffusion-dominated and Fair competitions regimes

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We analyse under which conditions equilibration between two competing effects, repulsion modelled by nonlinear diffusion and attraction modelled by nonlocal interaction, occurs. I will discuss several regimes that appear in aggregation diffusion problems with homogeneous kernels. I will first concentrate in the fair competition case distinguishing among porous medium like cases and fast diffusion like ones. I will discuss the main qualitative properties in terms of stationary states and minimizers of the free energies. In particular, all the porous medium cases are critical while the fast diffusion are not. In the second part, I will discuss the diffusion dominated case in which this balance leads to continuous compactly supported radially decreasing equilibrium configurations for all masses. All stationary states with suitable regularity are shown to be radially symmetric by means of continuous Steiner symmetrisation techniques. Calculus of variations tools allow us to show the existence of global minimizers among these equilibria. Finally, in the particular case of Newtonian interaction in two dimensions they lead to uniqueness of equilibria for any given mass up to translation and to the convergence of solutions of the associated nonlinear aggregation-diffusion equations towards this unique equilibrium profile up to translations as time tends to infinity. This talk is based on works in collaboration with S. Hittmeir, B. Volzone and Y. Yao and with V. Calvez and F. Hoffmann.

Some Geometrical and Physical Aspects of Fluid Motion: from Solitons to Stratified Flows

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We shall review and discuss various aspects of the theory of nonlinear integrable of near-to integrable PDEs, with a special focus on their Hamiltonian properties. In the first part we shall review the so-called Hamiltonian path to soliton equations with a special emphasy towards their conservation laws.

Then we shall "move away" from full integrability and turn towards the theory of incompressible stratified Euler fluids in 2D channels. Still the focus will be on conserved quantities, both for the continuous and the sharp (two-layer) stratifications. Building on works of Benjamin and Zakharov, the Hamiltonian
pictures will be discussed and specialized to our model(s). In particular, we shall show how to reduce the full 2D Hamiltonian structure to the 1D averaged long wave equations. We shall show that in the two-layer case the long wave model may finally be reduced to the Airy system (a/k/a/ dispersionless Non-Linear Schrödinger Equation). This happens provided the so-called Boussinesq approximation (retaining stratification only in the buoyancy terms) be enforced. The general non-Boussinesq case can be treated as a deformation of such an integrable system of PDEs.

This is a summary of joint works with F. Magri (Unimib), R.A. Camassa (UNC - Chapel Hill), G. Ortenzi (Unimib), M. Pedroni (Unibg) and others.

**Integrability in action: stability problems and the emergence of rogue waves**

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What is integrability? Amusingly, it is not easy to answer precisely. The question itself can occupy a whole book (Zakharov 1991), or, following Hitchin, Segal, Ward and Woodhouse, be dismissed as Louis Armstrong is believed to have done once when asked what jazz was - "If you gotta ask, you’ll never know!" In the same spirit, in this talk I will not define what an integrable system is, I will rather rely on one of the possible definitions and I will show how one can use integrability to answer very precise questions, such as the linear stability of nonlinear waves, or the necessary condition for the existence of certain solutions (rational solitons) possibly modelling coherent structure such as rogue waves.

The theory will be presented using the example of a system of two coupled nonlinear Schrödinger equations in the defocusing, focusing and mixed regimes. The derivation of the stability spectra is completely algorithmic and make use of elementary algebraic-geometry. It turns out indeed that, for a Lax Pair that is polynomial in the spectral parameter, the problem of classifying the spectra is transformed into a problem of classification of certain algebraic varieties. The method is general enough to be applicable to a large class of integrable systems.

This work has been done in collaboration with Antonio Degasperis (Roma "La Sapienza") and Matteo Sommacal (Northumbria University).
Overdetermined elliptic problems and constant mean curvature surfaces

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We present the classical result of J. Serrin (1971) about an overdetermined problem which has applications in several physics problem. The proof of J. Serrin was inspired by a PDE proof of the classical problem in geometry of characterizing the closed constant mean curvature surfaces, given by A.D. Alexandrov in 1962.

During the talk the connections between these two classical problems will be discussed, also in view of two other alternative proofs given by H. Weinberger and R. Reilly, based on integral identities.

Then we present recent developments for partially overdetermined problems and their connection with a variant of the classical Saint Venant Principle about the question of minimizing the torsional rigidity.

Liouville type theorems and local behaviour of solutions to degenerate or singular problems

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We consider an equation in divergence form with a singular/degenerate weight

\[-\text{div}(y^a A(x,y) \nabla u) = |y|^a f(x,y) \quad \text{or} \quad \text{div}(y^a F(x,y)),\]

Under suitable regularity assumptions for the matrix $A$ and $f$ (resp. $F$) we prove Hölder continuity of solutions and possibly of their derivatives up to order two or more (Schauder estimates). In addition, we show stability of the $C^{0,\alpha}$ and $C^{1,\alpha}$ a priori bounds for approximating problems in the form

\[-\text{div}((\varepsilon^2 + y^2)^a A(x,y) \nabla u) = (\varepsilon^2 + y^2)^a f(x,y) \quad \text{or} \quad \text{div}((\varepsilon^2 + y^2)^a F(x,y))\]

as $\varepsilon \to 0$. Finally, we derive $C^{0,\alpha}$ and $C^{1,\alpha}$ bounds for inhomogenous Neumann boundary problems as well. Our method is based upon blow-up and appropriate Liouville type theorems.
Optimal propagation of the support of solutions to anistropic evolution operators

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I will speak of a common paper with Düzgun and Mosconi. We consider a quasilinear parabolic Cauchy problem with spatial anisotropy of orthotropic type and study the spatial localization of solutions. Assuming the initial datum is localised with respect to a coordinate having slow diffusion rate, we bound the corresponding directional velocity of the support along the flow. The expansion rate is shown to be optimal for large times. We recall that many materials, such as liquid crystals, wood or earth’s crust usually present different diffusion rates along different directions. Moreover, in most of the physical phenomena involved in such media, finite speed of propagation of disturbances is a much more reasonable assumption than the usual infinite-speed one implied by linear equations. This effect can either be caused by additional absorption terms in the model equation or by the intrinsic diffusion rate of the medium. We are interested in this latter situation which, in the framework assumed here, consists in studying the evolution equation when some of the $p_i$’s are greater than 2.
Mean field equations and the global bifurcation diagram of the Gel’fand problem

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For a large class of domains \( \Omega \subset \mathbb{R}^2 \) (which need not be neither simply connected nor symmetric) we describe the qualitative behavior of the global bifurcation diagram of the unbounded branch of solutions crossing the origin \((\mu, v) = (0, 0) \in \mathbb{R} \times C^2_0(\Omega)\) of the Gel’fand problem,

\[
\begin{align*}
-\Delta v &= \mu e^v \text{ in } \Omega \\
v &= 0 \text{ on } \partial \Omega
\end{align*}
\]

After an introduction to some well known results, we will describe the main ideas behind the proof [1, 2]. In particular we will see that one can parametrize the solutions as a function of the energy of the associated mean field equation, naturally arising in the Onsager statistical description of 2-d turbulence. At least to our knowledge this is the first result about the exact monotonicity of the branch of non-minimal solutions which is not just concerned with radial solutions and/or with symmetric domains. This is part of a joint research project in collaboration with A. Jevnikar.

References


Liouville type results for a nonlocal obstacle problem

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My talk will be devoted to the qualitative properties of some nonlocal reaction-diffusion equations set on “perforated” open sets. One of the cornerstones in the study of this type of problem lies in suitable rigidity results of Liouville-type, which allow the classification of stationary solutions. I will give some results in this direction, under some geometric assumptions on the domain. This talk is based on some works with J. Coville, F. Hamel and E. Valdinoci.

References


Regularity and rigidity results for nonlocal minimal graphs

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Nonlocal minimal surfaces are hypersurfaces of Euclidean space that minimize the fractional perimeter, a geometric functional introduced in 2010 by Caffarelli, Roquejoffre & Savin in connection with phase transition problems displaying long-range interactions.

In this talk, I will introduce these objects, describe the most important progresses made so far in their analysis, and discuss the most challenging open questions.

I will then focus on the particular case of nonlocal minimal graphs and present some recent results obtained on their regularity and classification in collaboration with X. Cabré (ICREA & UPC, Barcelona), A. Farina (Université de Picardie, Amiens), L. Lombardini (UWA, Perth).
Muckenhoupt $A_p$-properties of distance functions and applications to Hardy-Sobolev type inequalities

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Let $X$ be a metric space equipped with a doubling measure. We consider weights $w(x) = \text{dist}(x,E)^{-\alpha}$, where $E$ is a closed set in $X$ and $\alpha \in \mathbb{R}$. We present sharp conditions, based on the Assouad (co)dimension of $E$, for the inclusion of $w$ in Muckenhoupt’s $A_p$ classes of weights, $1 \leq p < \infty$. We will also present an application of these results: (global) fractional Hardy-Sobolev inequalities in the setting of metric spaces. Joint work with Lizaveta Ihnatsyeva, Juha Lehrbäck, Heli Tuominen and Antti V. Vähäkangas.

Unique continuation principles for higher order fractional equations

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In this talk we discuss strong unique continuation principle and unique continuation from sets of positive measure for solutions of higher order fractional equations in open domains. By the Caffarelli-Silvestre extension method the problem is formulated as a system of two second order equations with singular or degenerate weights in a half-space, for which asymptotic estimates are derived by a blow-up analysis and energy estimates obtained by studying an Almgren type frequency function. The results discussed in the talk are contained in joint papers with A. Ferrero.

References


On the regularity of the minimizer of the electrostatic Born-Infeld energy and the existence of spacelike hypersurfaces of prescribed mean curvature in $\mathbb{L}^{N+1}$

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In this talk we show some new results concerning the regularity in $W^{2,2}_{\text{loc}}(\mathbb{R}^N)$, and in $C^{1,\alpha}_{\text{loc}}(\mathbb{R}^N)$, of the unique minimizer $u_\rho$ of the electrostatic Born-Infeld energy, which is

$$\int_{\mathbb{R}^N} \left( 1 - \sqrt{1 - |\nabla u|^2} \right) dx - \int_{\mathbb{R}^N} \rho u dx,$$

where $\rho \in L^m(\mathbb{R}^N)$ is an assigned charge density, $m \in [1, 2_\ast]$, $2_\ast := \frac{2N}{N+2}$, $N \geq 3$.

We also discuss and show that under suitable assumptions on $\rho$ the minimizer is a weak solution of the associated PDE

$$-\text{div} \left( \frac{\nabla u}{\sqrt{1 - |\nabla u|^2}} \right) = \rho \quad \text{in } \mathbb{R}^N, \quad (BI)$$

which is the equation for spacelike hypersurfaces of prescribed mean curvature in the Lorentz-Minkowski space $\mathbb{L}^{N+1}$, with the boundary condition $\lim_{|x| \to \infty} u(x) = 0$.

These results are collected in the works [1, 2], written in collaboration with D. Bonheure (Université Libre de Bruxelles).

References


Radial solutions of fully nonlinear elliptic equations in exterior domains

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We present some sharp existence/non existence results for radial solutions in exterior domains of fully nonlinear equations involving Pucci’s operators. In particular, we show the existence of solutions vanishing on the boundary and being fastly, slowly or pseudo-slowly decaying at infinity.

Mean-field and prescribed curvature problems - From local to nonlocal

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We consider a mean field equation of the form

\[
(−Δ)^n u = \rho \frac{e^{nu}}{\int_{\Omega} e^{nu} dx} \quad \text{in } \Omega \subset \mathbb{R}^n,
\]

under suitable boundary condition. When \( n \) is even, the problem was studied (among many others) by Suzuki [6] when \( n = 2 \) and in higher even dimension in [4, 5]. A very useful tool is the blow-up analysis, that naturally leads to the study of conformal metrics of constant Q-curvature in \( \mathbb{R}^n \), namely solutions to

\[
(−Δ)^n u = e^{nu} \quad \text{in } \mathbb{R}^n
\]

Recently, with DelaTorre, Hyder and Sire, we considered the 1-dimensional case (\( n = 1 \) and \( \Omega = (−1, 1) \)) of (1), giving existence and non-existence results. The novelty is that in this case (1) is nonlocal. When \( n \geq 3 \) is odd, even in the case of a ball, neither existence results, nor a blow-up analysis are known, but we expect that they would rest on asymptotics of solutions of (2), as studied in [2] or, with a conical singularity, in [3].

References


Elliptic systems with critical growth

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I will present some results concerning existence of sign-changing solutions to the Yamabe problem and their relation with positive solutions to a class of elliptic systems with critical growth.
Asymptotic spherical shapes in some spectral optimization problems

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We study the positive principal eigenvalue of a weighted problem associated with the Neumann Laplacian. This analysis is related to the investigation of the survival threshold in population dynamics. When trying to minimize such eigenvalue with respect to the weight, one is lead to consider a shape optimization problem, which is known to admit spherical optimal shapes only in very specific cases. We investigate whether spherical shapes can be recovered in general situations, in some singular perturbation limit. We also consider a related problem, where the diffusion is triggered by a fractional $s$-Laplacian, and the optimization is performed with respect to the fractional order $s \in (0,1]$. These are joint works with Dario Mazzoleni and Benedetta Pellacci.

References


Measure solutions to a system of continuity equations driven by Newtonian nonlocal interactions

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I will discuss a joint work with J.A. Carrillo, M. Di Francesco, S. Fagioli and M. Schmidtchen. In [2], we prove global-in-time existence and uniqueness of measure solutions of a nonlocal interaction system of two species in one spatial dimension. For initial data including atomic parts we provide a notion of gradient-flow solutions in terms of the pseudo-inverses of the corresponding cumulative distribution functions, for which the system can be stated as a gradient flow on the Hilbert space $L^2(0,1)^2$ according to the classical theory by Brézis [1]. For absolutely continuous initial data we construct solutions using a minimising movement scheme in the set of probability measures. In addition we show that the scheme preserves finiteness of the $L^m$-norms for all $m \in (1, +\infty]$ and of the second moments. We then provide a characterisation of equilibria and prove that they are achieved (up to time subsequences) in the large time asymptotics. We conclude the paper constructing two examples of non-uniqueness of measure solutions emanating from the same (atomic) initial datum, showing that the notion of gradient flow solution is necessary to single out a unique measure solution.

References


The deterministic particle limit towards a nonlocal interaction equation with nonlinear mobility

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In many applied contexts in biology and real world applications, nonlocal aggregation phenomena are better described by the PDE

\[ \partial_t \rho = \text{div}(\rho v(\rho) \nabla W * \rho), \quad \rho = \rho(x,t), \quad x \in \mathbb{R}^d, \quad t \geq 0, \]

where \( W : \mathbb{R}^d \to \mathbb{R} \) is an aggregation kernel and \( v : [0, +\infty) \to [0, +\infty) \) is a decreasing velocity profile such that \( v(0) = v_{\text{max}} > 0 \) and \( v(\rho_{\text{max}}) = 0 \) for some maximal density \( \rho_{\text{max}} > 0 \). The function \( v \) accounts for overcrowding preventing effects inhibiting the emergence of blow-up in finite time.

We will describe a recent result in [1], in which the solution \( \rho \) to the above problem (coupled with a nonnegative initial condition in \( BV(\mathbb{R}) \)) is approximated rigorously by the empirical measure of a suitable nonlocal version of a follow-the-leader particle scheme, in which particles move according to certain ODE’s. A similar results was proven for a homogeneous scalar conservation law in [2]. We will prove that the scheme provides convergence towards the unique entropy solution to the equation [3], and we will explain why entropy solutions are suitable in this context.

References


Nonlinear diffusions on manifolds

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We discuss asymptotics of solutions to the porous medium equation on negatively curved manifolds. The results depend crucially on the growth assumptions on curvature at infinity. In fact if curvature is superquadratic a surprising connection with an associated elliptic problem, which is shown to have a solution in that case, arise.

References


Higher integrability for doubly nonlinear parabolic equations

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We discuss a local higher integrability result for the spatial gradient of weak solutions to doubly nonlinear parabolic equations of the type

\[(|u|^{p-2}u)_t - \text{div}(|Du|^{p-2}Du) = 0\]

in the range

\[\max\left\{ \frac{2n}{n+2}, 1 \right\} < p < \frac{2n}{(n-2)_+},\]

where \(n \in \mathbb{N}\) is the spatial dimension. In [1] we show that a gradient of a nonnegative weak solution to a doubly nonlinear equation belongs locally to a slightly higher Sobolev space than assumed a priori with a
reverse Hölder type estimate. The range may seem unexpected, but the lower bound also appears in the
higher integrability for the parabolic $p$-Laplace equation [6], while the upper bound is the same as for the
porous medium equation in the fast diffusion range [5] and [3].

The equation is homogeneous in the sense that solutions are invariant under multiplication by constants,
but constants cannot be added without destroying the property of being a solution. The key ingredient in
the proof of our main result is an appropriate intrinsic geometry that depends on the the solution as well
as its spatial gradient and thus allows us to rebalance the mismatch between the function and its gradient
in the equation. Variants of this technique have been successfully used in proving the corresponding results
for the parabolic $p$-Laplace equation [6] and for the porous medium equation [4], [5], [2] and [3]. Related
results and open questions are also discussed.

References


Dynamic formulation of the optimal transport distance and some applications to Bayesian stability

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Within the optimal transport theory, we consider families of probability measures that arise as regular conditional distributions parametrised by the observable value, and we study their uniform continuity with respect to the Wasserstein distance. Our approach is based on the dynamical formulation of the optimal transport distance and leads to analyse continuity equations with unknown velocity vector fields. The dynamical approach leads to stability results in Bayesian statistical inference, with specific focus on exponential models and Pareto models.
This is a joint work with Emanuele Dolera.

**Blow-up vs. global existence for the porous medium equation on certain Cartan-Hadamard manifolds**

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We consider the porous medium equation (PME) with a power-type reaction term, posed on complete, simply-connected Riemannian manifolds $M$ having everywhere nonpositive sectional curvatures, also known as Cartan-Hadamard manifolds. More precisely, we study the following problem:

$$ \begin{cases} u_t = \Delta(u^m) + u^p & \text{in } M \times (0, T), \\ u = u_0 & \text{in } M \times \{0\}, \end{cases} $$

where $m, p > 1$, $T \in (0, +\infty]$ and $u_0$ is a nonnegative, bounded, compactly-supported initial datum. We suppose in addition that the Ricci curvature of $M$ is bounded either from above or from below by a negative constant, the model case being represented by the hyperbolic space $\mathbb{H}^n$. It turns out that if $p > m$ small data give rise to solutions that exist globally in time, while solutions associated to sufficiently large data blow up in finite time, at least in the $L^\infty(M)$ sense. If $p < m$ we can only assert that solutions starting from sufficiently large data blow up at worst in infinite time, i.e. $T$ may be $T = +\infty$ but $\|u(t)\|_{\infty}$ does not stay globally bounded, whereas under the stronger requirement $p \in (1, (1 + m)/2]$ we can ensure that finite-time blow up never occurs, i.e. solutions exist globally ($T$ is indeed $+\infty$). Our strategy of proof relies on pure barrier methods, modeled after the ones provided by [4] for the standard PME and having in mind the Euclidean techniques set up in [1, 2]. The results we obtain are in several aspects different from the Euclidean ones, as has to be expected because negative curvature is known to produce a diffusion-acceleration phenomenon.

The content of the talk is based on the paper [3].

**References**


Estimates for solutions to Cauchy problem for a class of nonlinear degenerate parabolic equations

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We consider a Cauchy problem associated to the following class of nonlinear degenerate parabolic equations whose prototype is the parabolic p-Laplacian ($p > 2$)

$$\frac{\partial u}{\partial t} - \text{div} A(x,t,u,Du) = 0, \quad \text{in } \mathbb{R}^N \times (0, \infty)$$

with

$$A = (A_1, \ldots, A_N) : \mathbb{R}^N \times (0, \infty) \times \mathbb{R} \times \mathbb{R}^N \rightarrow \mathbb{R}^N$$ is a Caratheodory function, i.e.

the mapping $\mathbb{R}^N \times (0, \infty) \ni (x,t) \mapsto A(x,t,u,\xi)$ is measurable $\forall u \in \mathbb{R}$ and $\xi \in \mathbb{R}^N$,

$\mathbb{R} \times \mathbb{R}^N \ni (u, \xi) \mapsto A(x,t,u,\xi)$ is continuous for almost all $(x,t) \in \mathbb{R}^N \times (0, \infty)$.

Under suitable "ellipticity and growth conditions" on $A$, we establish sharp pointwise upper and lower estimates for nonnegative solutions to the problem under investigation. Then we use these results to prove existence and sharp pointwise upper and lower estimates for the fundamental solutions.

This is a joint research with V. Vespri e F. Ragnedda.

References


Long–time asymptotics for a 1D nonlocal porous medium equation with absorption or convection

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In this talk, we show some results about the long-time asymptotic behaviours of one-dimensional porous medium equations with a fractional pressure and absorption or convection are. In the parameter regimes when the nonlocal diffusion is dominant, the entropy method is suitably adapted to derive the exponential convergence of relative entropy of solutions in similarity variables. These results are objects of the joint work [1].

References

[1] F. Feo, Y. Huang, B. Volzone, Long-time asymptotics for a 1D nonlocal porous medium equation with absorption or convection, 2019, in press.
Breathing soliton dynamics in mode-locked fibre lasers

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Dissipative solitons (DSs) in a nonlinear medium are localised coherent structures that result from the composite balance between conservative effects (nonlinearity and dispersion/diffraction) and dissipative ones (gain and loss) \cite{1}. In addition to parameter-invariant stationary DSs, numerous nonlinear systems support breathing (pulsating) DSs, the energy of which is localised in space but oscillates in time, or vice versa \cite{3}. Such nonlinear waves are attracting considerable research interest in optics owing to their strong connection with the Fermi-Pasta-Ulam paradox, formation of rogue waves, turbulence and modulation instability phenomena. Apart from their fundamental importance in nonlinear science, breathing solitons are also attractive because of their potential for practical applications, such as in spectroscopy. Yet, the observation of these breathers has been mainly restricted to optical microresonator platforms \cite{2}.

In this talk, I will report on the generation and study of breathing DSs in passively mode-locked fibre lasers \cite{4}. Breathing solitons feature periodic spectral and temporal evolutions over cavity round trips. Experimentally, we capture such fast dynamics spectrally and temporally in real time using time-stretch dispersive Fourier transform based single-shot spectral measurements \cite{5} and spatio-temporal intensity measurements. Remarkably, in the normal-dispersion regime of the laser cavity, breathers are excited in the laser under the pump threshold for stationary DS mode locking. For the first time in experiments with mode-locked fibre lasers, breathing soliton pair molecules are also generated in the cavity, which represent double-breather bound states with a close intra-pulse separation. The universal nature of the breather formation is indicated by our observation in a varying-length cavity, and further confirmed by numerical simulations of the laser model described by the complex cubic-quintic Ginzburg-Landau equation (CQGLE) \cite{1}. When the laser has an average anomalous cavity dispersion, we observe a regime of operation where the laser oscillator generates multiple pulsating solitons with extreme ratios of maximal to minimal intensities in each period of pulsations. The soliton spectra also experience large periodic broadening and compression. These observations are, to the best of our knowledge, the first of their kind in a laser system.

Breathers introduce a new regime of mode locking into ultrafast lasers. These findings not only carry importance from an application perspective, but also contribute more broadly to the fundamental understanding of dissipative soliton physics. Our observations further demonstrate that mode-locked fibre lasers are an ideal test bed for the study of complex nonlinear wave dynamics relevant to a large variety of physical systems. More generally, the complex CQGLE is the most common mathematical implementation of a dissipative system, describing many different nonlinear effects in physics, such as nonlinear waves, superconductivity, superfluidity, Bose-Einstein condensates, liquid crystals, plasmas, and numerous other
phenomena. Therefore, it is reasonable to assume that the breathing DS dynamics found in this work are not limited to optical systems and will also be discovered in various other physical systems.

References


Hydrodynamic models and boundary confinement effects

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Confinement effects by rigid boundaries in the dynamics of ideal fluids are considered from the perspective of long-wave models and their parent Euler systems, with the focus on the consequences of establishing contacts of material surfaces with the confining boundaries. When contact happens, it can be shown that the model evolution can lead to the dependent variables developing singularities in finite time. The conditions and the nature of these singularities are illustrated in several cases, progressing from a single layer homogeneous fluid with a constant pressure free surface and flat bottom, to the case of a two-fluid system contained between two horizontal rigid plates, and finally, through numerical simulations, to the full Euler stratified system. These illustrate the qualitative and quantitative predictions of the models within a set of examples chosen to illustrate the theoretical results.
Large–time asymptotic stability of travelling wave solutions to scalar balance laws

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We prove the large-time asymptotic orbital stability under piecewise regular perturbations for classes of solutions to first order scalar hyperbolic balance laws:

- Riemann shocks, provided that the source term is dissipative about endstates of the shock;
- smooth fronts connecting two dissipative endstates, through a sonic point defining its velocity.

Any other “generic” travelling wave solution is unstable. This is a joint work with Luis Miguel Rodrigues.

Tau functions of the Drinfeld–Sokolov hierarchies and Schur polynomials

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The Drinfeld–Sokolov hierarchies are families of integrable partial differential equations that are constructed via an affine Kac-Moody algebras. The tau function is one single function that encodes a solution to a given hierarchy. In the spirit of Sato’s expansion of the tau function of the KP hierarchy in terms of Schur polynomials and Plucker coordinates, I will show how to expand the tau function of any Drinfeld-Sokolov hierarchy over integer partitions via a natural generalization of Schur polynomials for arbitrary semisimple Lie algebras. We make use of the large-size limit Toeplitz determinants and the Sato-Segal-Wilson Grassmannian approach. We deduce a simple criterion for the polynomiality of the tau function. This is a joint work with Mattia Cafasso (Université d’Angers, France) and Di Yang (USTC, China).
Integrability in 3D

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We consider the problem of detecting and classifying integrable partial differential (and difference) equations in 3D. Our approach is based on the observation that dispersionless limits of integrable systems in 3D possess infinitely many multi-phase solutions coming from the so-called hydrodynamic reductions. We consider a novel perturbative approach to the classification problem of dispersive equations. Based on the method of hydrodynamic reductions, we first classify integrable quasilinear systems which may (potentially) occur as dispersionless limits of soliton equations in 3D. To reconstruct dispersive deformations, we require that all hydrodynamic reductions of the dispersionless limit are inherited by the corresponding dispersive counterpart. This procedure leads to a complete list of integrable third and fifth order equations, which generalize the examples of Kadomtsev-Petviashvili, Veselov-Novikov and Harry Dym equations as well as integrable Davey-Stewartson type equations, some of which are apparently new. We also consider the problem of dispersive deformations on the Lax representation level and thus show that our approach allows starting from the dispersionless Lax representations to construct the fully dispersive Lax pairs representing the fully dispersive integrable systems. We extend this approach to the fully discrete case. Based on the method of deformations of hydrodynamic reductions, we classify discrete 3D integrable Hirota-type equations within various particularly interesting subclasses as well as a number of classification results of scalar differential-difference integrable equations including that of the intermediate long wave and Toda type.

Evolution of a two-layer fluid and detachment of its interface from horizontal boundaries

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In this talk I will present some features of the evolution of a 2-dimensional stratified fluid in a channel with horizontal boundaries. After recalling that, despite the translational invariance of the system, total horizontal momentum is not conserved, I will consider an air-water system (i.e., the case where the density of the upper fluid is zero) and I will show that the connection properties of the air domain affect total horizontal momentum conservation. Then I will discuss, in the cases of a single and of a two-layer fluid, if and how an interface initially touching the bottom (or the top) of the channel can evolve in such a way to detach from the boundaries.
These results have been obtained in collaboration with Roberto Camassa, Gregorio Falqui, Giovanni Ortenzi, and others.

**Heisenberg ferromagnetism as an evolution of a spherical indicatrix: localized solutions and elliptic dispersionless reduction**

*Matteo Sommacal*

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The connection between the focusing nonlinear Schrödinger equation (NLS) and the continuous, classical Heisenberg ferromagnet equation (HF) is investigated through the Da Rios model (DR). The binormal equation describing the motion of a vortex filament in an incompressible Euler fluid with constant density is derived under the local induction approximation hypotheses. The resulting filament curvature and torsion, which satisfy the Da Rios system, can be given in terms of the solutions of NLS by means of the Hasimoto map. At the same time, the unitary tangent vector to the vortex filament defined as a solution of the binormal equation can be identified with a solution of HF. This allows a two-way implementation of the Zakharov-Takhtajan gauge equivalence, which links the inverse scattering transform (IST) spectral parameters of NLS to the spectral description of HF.

Given a vortex filament, the solutions of HF are represented by the corresponding spherical (or tangent) indicatrix. The equations for the time evolution of the indicatrix on the unit sphere are given explicitly in terms of the geodesic curvature and the arclength of the curve. Notably, by neglecting the dispersive terms, a novel dispersionless reduction of the Heisenberg ferromagnet model is obtained. The length of the spherical indicatrix is proved not to be conserved.

Finally, a totally explicit algorithm is provided, allowing to construct a solution of HF from a solution of NLS, and, remarkably, vice versa, allowing to construct a solution of NLS from a solution of HF. In the reflectionless case, such a two-way map between solutions is shown to preserve the IST spectra, and thus the localization.

This research has been carried out in collaboration with Francesco Demontis (Università di Cagliari) and Giovanni Ortenzi (Università di Milano Bicocca).
After a quick review of the direct and inverse scattering theory of the focusing Zakharov-Shabat system with symmetric nonvanishing boundary conditions, we derive the exact expressions for its reflectionless solutions using Marchenko theory. Since the Marchenko integral kernel has separated variables, the matrix triplet method consisting of representing the Marchenko integral kernel in the form

$$F_r(x+y,t) = C e^{-(x+y)A} e^{tH} B$$

is applied to express the multisoliton solutions of the focusing nonlinear Schrödinger equation with symmetric nonvanishing boundary conditions in terms of the matrix $(A, B, C)$. Since these exact expressions contain matrix exponentials and matrix inverses, computer algebra can be used to “unpack” and graph them. Here $A$ has only eigenvalues with positive real part, $H$ is a suitable function of $A$, and $B$ and $C$ are size compatible rectangular matrices. The $2p \times 2q$ matrices involved are $p \times q$ matrices with its entries belonging to a division ring of $2 \times 2$ matrices that is isomorphic with Hamilton’s quaternion algebra, thus supplying an application of quaternion linear algebra [1]

References

The role of a strong confining potential in a nonlinear Fokker–Planck equation

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Solutions of nonlinear nonlocal Fokker–Planck equations in a bounded domain with no-flux boundary conditions can be approximated by Cauchy problems with increasingly strong confining potentials defined in the whole space. Two different approaches are analyzed, making crucial use of uniform estimates for $L^2$ energy functionals and free energy (or entropy) functionals respectively. In both cases, the weak formulation of the problem in a bounded domain can be obtained as the weak formulation of a limit problem in the whole space involving a suitably chosen sequence of large confining potentials. The free energy approach extends to the case degenerate diffusion.

This is joint work with M. Bruna and J. A. Carrillo, [1].

References


Holder Continuity of Solutions for an Anisotropic $p$-Laplacian Equation by Parabolic Approach

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We consider locally bounded weak solutions to the quasilinear anisotropic equation

\[
\frac{\partial}{\partial x} A_p(z,u,Du) + \sum_{i=1}^{n-1} \frac{\partial}{\partial y_i} A_{q_i}(z,u,Du) = B(z,u,Du)
\]  

(1)

where $A_p$ operator has $p$-growth in $x$ direction and $A_{q_i}$ operators have $q$-growth in $y_i$ direction. The prototype equation for this operator is the orthotropic $(p,q)$-Laplacian.
\[
\frac{\partial}{\partial x} \left( \frac{\partial}{\partial x} u \right)^{p-2} \frac{\partial}{\partial x} u + \sum_{i=1}^{n-1} \frac{\partial}{\partial y_i} \left( \frac{\partial}{\partial y_i} u \right)^{q-2} \frac{\partial}{\partial y_i} u = 0
\]

with \( z = (x,y) \in \Omega \subset \mathbb{R}^n \) open and bounded, \( 1 < p \leq q < \infty \). The interest in these problems is related to a huge number of applications ranging from modeling electro-rheological fluids, image processing to the theory of elasticity, as well as mathematical reasons: indeed modifications to the classical methods are needed because of non-linearity and non-homogeneous nature of operators. We demonstrate Holder Continuity by applying an adaptation to the anisotropic case of the intrinsic scaling method originally used by Di Benedetto for parabolic equations. Intrinsic cylinders are natural domains where to consider local solutions, as suggested by physical interpretation of the models and by the structure of the equations. If the coefficients are merely bounded and measurable, Hölder continuity has been established in [5] in the special case of \( p = 2 \) and expansion in positivity for the homogeneous case has been developed in [2]. A dual result has been proved recently in [3]. The main idea is to regard the equation as parabolic with respect to the variable \( x \), corresponding to \( p \), and to apply the techniques of [4]. When positivity is clustered aside, we expand it through a logarithmic Lemma and a double change of variables, which is needed to reestablish the correct intrinsically scaled geometry. Finally, when positivity weights enough almost everywhere, the techniques originally introduced by E. De Giorgi in [1] are employed to get reduction of the oscillation. This approach is only measure-theoretical in nature and as such holds the promise of a wider applicability.

References


Photometric Stereo under unknown lights position

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A classical problem in Computer Vision consists in reconstructing the 3D shape of an object, starting from a set of digital pictures. Photometric Stereo is a photographic technique used to extract shape and color information from an object which is observed from the same fixed point of view but under different lighting conditions.
lighting conditions. We will describe an algorithm to approximate the framed object, treating, in particular, the more intricate case where the position of the light sources is unknown. Various attempts have been made to estimate the lights position directly from the data; see, e.g., [1, 2] where a linear combination of special functions (spherical harmonics) is employed. Obtaining such a result would release the constraint on the precise positioning of the light sources, making the surveying process much simpler. Numerical experiments will be illustrated, showing that the light sources position can be estimated directly from the data when at least 6 pictures of the observed object are available, under suitable conditions.

This work is a joint research with R. Dessì, C. Fenu, G. Rodriguez and M. Vanzi.

References


Electromagnetic data inversion through a linear integral model: existence, uniqueness and numerical approximation of solutions

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This work investigates an integral linear model, which describes the interaction of an electromagnetic field with the soil, in order to recover the electrical conductivity of the soil.

After we prove the existence and uniqueness of solutions, we propose different numerical methods to approximate such solution: these approximation procedures are based on piecewise constant functions, linear splines, and Bernstein polynomials combined with some suitable regularization techniques.

Finally we test the effectiveness of the different numerical methods on synthetic data sets.

References


Fractional weighted eigenvalue problems: monotonicity and optimization

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We consider the weighted eigenvalue problem for a general non-local pseudo-differential operator, depending on a bounded weight function. For such problem, we prove that strict monotonicity of the eigenvalues with respect to the weight function is equivalent to the unique continuation property of eigenfunctions.

In the particular case of the fractional Laplacian we study the optimization of the map \( \rho \mapsto \lambda_k(\rho) \), where \( \lambda_k \) is the \( k \)-th eigenvalue and the weight \( \rho \in \mathcal{G} \), where \( \mathcal{G} \) is a set of equimeasurable functions. We show the existence of minimizers and maximizers in the closure \( \overline{\mathcal{G}} \) of \( \mathcal{G} \) in \( L^\infty(\Omega) \) with the weak* topology. In particular, for \( k = 1 \) we prove that minimizers are in \( \mathcal{G} \) and that, if \( \Omega \) is Steiner symmetric, they also have the same symmetry.

References


Properties of solutions to some reaction-diffusion-taxis problems

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We study properties of nonnegative and classical solutions to reaction-diffusion-taxis problems defined in bounded and smooth domains of \( \mathbb{R}^N \), \( N \geq 1 \). In particular we dedicate to chemotaxis and porous medium models, focusing our attention on global and local solutions. Further, for those situations where blow-up occurs, we estimate such blow-up time.
References


Weak solution for Neumann \((p,q)\)-Laplacian problem on Riemannian manifold

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We prove the existence of a nontrivial solution for a nonlinear \((p,q)\)-Laplacian problem with Neumann boundary condition, on a non compact Riemannian manifold. The idea is to reduce the problem in variational form, which means to consider the critical points of the corresponding Euler–Lagrange functional in an Orlicz–Sobolev space.

References


Solving nonlinear problems: different approaches to regularization

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Abstract

Let us assume that \(F(x)\) is a nonlinear Fréchet differentiable function, with value in \(\mathbb{R}^m\) for any \(x \in \mathbb{R}^n\). For a given \(b \in \mathbb{R}^m\), we solve the least squares problem \(\min_x \frac{1}{2} \|r(x)\|^2\), where \(r(x) = F(x) - b\) is the residual vector function, by applying both Newton’s and Gauss–Newton methods [1]. Newton’s method is based on a second-order Taylor approximation of the function \(f(x) = \frac{1}{2} \|r(x)\|^2\), and subsequent minimization of the resulting approximate function. The Gauss–Newton method is based on a sequence of linear approximations of \(r(x)\) obtained through the Taylor series.
The nonlinear function $F(x)$ is considered ill-conditioned in a domain $\mathcal{D} \subset \mathbb{R}^n$ when the condition number $\kappa(J)$ of the Jacobian $J = J(x)$ of $F(x)$ is very large for any $x \in \mathcal{D}$. It may also happen that, during the iterations of Gauss–Newton method, the matrix $J$ becomes rank-deficient. Under this assumption, it is common to apply a regularization method to each step of the Gauss–Newton method. We compare this situation to applying the same regularization method to the initial nonlinear least squares problem. We apply these two approaches to a geophysical model used for electromagnetic data inversion [2, 3].

This work is a joint research with G. Rodriguez.

References


# Program of the Meeting

All the invited speakers will present their contributions at the *Hotel Flamingo Resort*, according to the following program.

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