Majorization and its applications in quantum information

Corso di Dottorato in Filosofia, Epistemologia, Scienze Umane, Università di Cagliari

Visiting Professor: Prof. Gustavo Martin Bosyk (Contact: gbosyk@gmail.com).

Coordinator: Prof. Giuseppe Sergioli (Contact: giuseppe.sergioli@gmail.com)

Chronogram: 4 lessons of 2hs. Starting on 9 May 15hs-17hs, and continues every Tuesday until 31 May, 2023 (Room: aula 11).

Teaching consultation meetings: 10/5, 17/5,22/5 and 30/5 from 15hs a 17hs (Room: Sergioli's office)

Room: Aula 11, Edificio Centrale, Dipartimento di Pedagogia, psicologia, filosofia, Università degli Studi di Cagliari

Short description and program:

In how many ways can one represent a given quantum mixed state as a mixture of pure states? Why (and in which sense) are separable states more disordered globally than locally? Is it possible to transform a given pure state into another by means of local operations and classical communication? How much entangled has a multipartite quantum state? How should an adequate formulation of the uncertainty principle be? All these questions, as dissimilar as they may seem, share one element in common: They can be answered by appealing to the notion of *majorization partial order*.

In this course, we attempt to make a brief review of the majorization theory and then to highlight the most important results of this research line in the quantum realm. In particular, we present and discuss a variety of situations to show that the spread applicability of majorization in the quantum realm emerges as a consequence of deep connections among majorization, partially ordered probability vectors, unitary matrices, and the probabilistic structure of quantum mechanics.

The program of course is as follows:

Part 1: Majorization theory. Definition and basic properties of majorization between probability vectors. Lorenz curve. Doubly stochastic matrices. Schur-concave functions and generalized entropies. Order-theoretic properties of majorization.Hermitian matrices and the Schur-Horn theorem.

Part 2: Quantum mechanics. Review of mathematical formalism (Dirac bra-ket notation). Postulates of quantum mechanics. Quantum states: pure and mixed states. Measurements: projective and generalized measurement. Probabilities: Born's rule. Quantum maps. Composite systems: Global and reduced density operator. Partial trace. Schmidt decomposition.

Part 3: Applications. Schrödinger mixture theorem. Quantum entropies. Majorization separability criteria. LOCC paradigm. Quantum teleportation. Nielsen theorem. Entanglement measures. Majorization uncertainty relations.

Bibliograpghy:

- A.W. Marshall, I. Olkin, B. Arnold, *Inequalities: Theory of Majorization and Its Applications*, 2ed., (Springer Verlag, New York City, 2011).
- M.A. Nielsen, G. Vidal, *Majorization and the interconversion of bipartite states*, Quantum Inf. Comput. 1 76, (2001).
- M. Nielsen, I. Chuang, *Quantum Computation and Quantum Information: 10th Anniversary Edition*, (Cambridge University Press, 2010).
- G. Bellomo, G. M. Bosyk, *Majorization, across the (quantum) universe*, (Cambridge University Press, 2019).