From quantum structures to quantum logics

PhD course

Davide Fazio

Generalities

Title: **From quantum structures to quantum logics**.

Teacher: **Davide Fazio**

Duration: **12 hours (5 classes + exam session)**.

The course will be held online on the Google Meet platform. The link will be made available one week before the beginning of the course.

Description

The course is a gentle introduction to quantum logics developed within the logico-algebraic approach to quantum mechanics. Starting from motivations contained in the seminal work by G. Birkhoff and J. von Neumann's "The logic of Quantum mechanics" [2], we will introduce orthomodular quantum logic arising in *sharp Quantum Theory*. Particular attention will be paid to order theoretical and algebraic properties of its equivalent algebraic semantics, namely orthomodular lattices. An order-theoretical analysis of the notion of contestuality will be offered by introducing the concept of a *pasting* of Boolean algebras.

Classes will be held in english or in Italian, depending on the audience.

Prerequisites

Basic competencies in universal algebra and order theory are required.

Final Exam

Students will give (individually or in small groups) final reading seminars on some of the latest research papers concerning the topics of the course.

Calendar and synopsis

- **Thursday 27th April 2023 from 17:00 to 19:00**. Motivations behind quantum logic: Birkhoff and von Neumann's approach to quantum theory. Orthomodular lattices: abstract definition, examples and well known constructions.
- **Wednesday 03rd May 2023 from 09:00 to 11:00.** Orthomodular lattices: basic algebraic and order theoretical properties (commutativity, perspectivity and structure theory) - Part I.
- **Friday 05th May 2023 from 09:00 to 11:00.** Orthomodular lattices: basic algebraic and order theoretical properties (commutativity, perspectivity and structure theory) - Part II.
- **Monday 08th May 2023 from 09:00 to 11:00.** Pastings of Boolean algebras, part I: Amalgam's of Boolean algebras and Greechie's theorems.
- **Friday 12th May 2023 from 09:00 to 11:00.** Pastings of Boolean algebras, part II: Dichtl's theorems.
- **Monday 22nd May 2023 from 09:00 to 11:00.** Students' final seminars.

References

[1] Beran L., *Orthomodular Lattices: Algebraic Approach*, Riedel, Dordrecht, 1985.

[2] Birkhoff G., von Neumann J., "The logic of quantum mechanics", *Annals of Mathematics*, 37, 1936, pp. 823-843.

[3] Dalla Chiara M. L., Giuntini R., Greechie R., *Reasoning in Quantum Theory–Sharp and Unsharp Quantum Logic*, Kluwer Dordrecht, 2004.

[4] Dichtl M.,"Astroids and pastings", *Algebra Universalis*, 18, 1984, pp. 380-385.