Doctoral Course 2023

Title: *Majorization theory and its applications to quantum information*

Professor: Dr. Gustavo Martin Bosyk

Hours: 8hs

Schedule: 4 lessons of 2hs. Starting on May, 2023. Schedule to be agreed with those interested.

Modality: presencial

Short description and program:

In how many ways can one represent a given quantum mixed state as a mixture of pure states? Why (and in which sense) are separable states more disordered globally than locally? Is it possible to transform a given pure state into another by means of local operations and classical communication? How much entangled has a multipartite quantum state? How should an adequate formulation of the uncertainty principle be? All these questions, as dissimilar as they may seem, share one element in common: They can be answered by appealing to the notion of majorization partial order.

In this course, we attempt to make a brief review of the majorization theory and then to highlight the most important results of this research line in the quantum realm. In particular, we present and discuss a variety of situations to show that the spread applicability of majorization in the quantum realm emerges as a consequence of deep connections among majorization, partially ordered probability vectors, unitary matrices, and the probabilistic structure of quantum mechanics.

The program of course is as follows:

Part 1: Majorization theory. Definition and basic properties of majorization between probability vectors. Lorenz curve. Doubly stochastic matrices. Schur-concave functions and generalized entropies. Order-theoretic properties of majorization.Hermitian matrices and the Schur-Horn theorem.

Part 2: Quantum mechanics. Review of mathematical formalism (Dirac bra-ket notation). Postulates of quantum mechanics. Quantum states: pure and mixed states. Measurements: projective and generalized measurement. Probabilities: Born's rule. Quantum maps. Composite systems: Global and reduced density operator. Partial trace. Schmidt decomposition.

Part 3: Applications. Schrödinger mixture theorem. Quantum entropies. Majorization separability criteria. LOCC paradigm. Quantum teleportation. Nielsen theorem. Entanglement measures. Majorization uncertainty relations.

Preliminary knowledge: it is needed elementary notions of linear algebra and it is preferable (but not necessary) basic notions of quantum mechanics.

Bibliography

M. Nielsen, I. Chuang, *Quantum computation and quantum information*, Cambridge University Press (2000).

I. Bengtsson, K. Życzkowski, *Geometry of Quantum States: An Introduction to Quantum Entanglement*, Cambridge University Press (2017).

Marshall, A. W., Olkin, I. and Arnold, B. (2010). Inequalities: Theory of Majorization and Its Applications. New York: Academic Press.

Nielsen, M. A. and Vidal, G. (2001). Majorization and the interconversion of bipartite states. Quantum Information & Computation, 1: 76-93.

Bellomo, G., Bosyk, G. M. (2019). Majorization, across the (quantum) universe.

Language: English.