PhD course

Davide Fazio

Generalities

Title: **From quantum structures to quantum logics**.

Teacher: **Davide Fazio**

Duration: **20 hours (9 classes)**.

The course will be held in presence or, due to eventual needs, online on the Microsoft Teams platform.

Description

The course is a gentle introduction to quantum logics developed within the logico-algebraic approach to quantum mechanics. Starting from motivations contained in the seminal work by G. Birkhoff and J. von Neumann's "The logic of Quantum mechanics" [2], we will introduce orthomodular quantum logic arising in *sharp Quantum Theory*. Particular attention will be paid to order theoretical and algebraic properties of its equivalent algebraic semantics, namely orthomodular lattices. Subsequently, we will introduce the class of orthomodular posets as a generalization of orthomodular lattices starting from their physical motivation, namely the axiomatic approach to sharp quantum theory. An order-theoretical analysis of the notion of contestuality will be offered by introducing the concept of a *pasting* of Boolean algebras. A discussion on the pasting construction of orthoalgebras will follow. Finally, we will consider several proof (Gentzen and Hilbert style) systems for orthomodular quantum logic. Their analysis will be expedient for setting orthomodular lattices in the landscape of residuated structures.
Classes will be held in english.

Prerequisites

Basic competencies in universal algebra and order theory are required.

Final Exam

Students will give (individually or in small groups) final reading seminars on some of the latest research papers concerning the topics of the course.

Calendar and synopsis

- **Friday 4th March 2022 from 14:00 to 16:00**. Motivations behind quantum logic: Birkhoff and von Neumann's approach to quantum theory. Orthomodular lattices: abstract definition, examples and well known constructions.
- **Friday 11th March 2022 from 14:00 to 15:30.** Orthomodular lattices: basic algebraic and order theoretical properties (commutativity, perspectivity and structure theory) - Part I.
- **Friday 18th March 2022 from 14:00 to 16:00.** Orthomodular lattices: basic algebraic and order theoretical properties (commutativity, perspectivity and structure theory) - Part II.
- **Friday 25th March 2022 from 14:00 to 16:00.** Orthomodular posets and their motivation: MacKey axiomatization of Quantum Theory. Orthomodular posets arising from concurrent systems, relational orthomodular posets.
- **Wednesday 31st March 2022 from 14:00 to 16:00.** Pastings of Boolean algebras, part I: Amalgam's of Boolean algebras and Greechie's theorems.
- **Friday 08th April 2022 from 14:00 to 16:00.** Pastings of Boolean algebras, part II: Dichtl's theorems.
- **Friday 15th April 2022 from 14:00 to 16:00.** Orthomodular lattices and their proof theory: Kalmbach's Hilbert system, Nishimura's proof system for orthologic and orthomodular quantum logic.
- **Friday 22nd April 2022 from 14:00 to 16:00.** Quantum logic vs. substructural logics: orthomodular lattices and residuation.
- **Friday 29th April 2022 from 14:00 to 18:00.** Students' final seminars.

References

[1] Beran L., *Orthomodular Lattices: Algebraic Approach*, Riedel, Dordrecht, 1985.

[2] Birkhoff G., von Neumann J., "The logic of quantum mechanics", *Annals of Mathematics*, 37, 1936, pp. 823-843.

[3] Dalla Chiara M. L., Giuntini R., Greechie R., *Reasoning in Quantum Theory–Sharp and Unsharp Quantum Logic*, Kluwer Dordrecht, 2004.

[4] Dichtl M.,"Astroids and pastings", *Algebra Universalis*, 18, 1984, pp. 380-385.