RESTRICTING FREE MV-ALGEBRAS WITH PRODUCT TO POLYHEDRA

SERAFINA LAPENTA

One of the main results in the theory of MV-algebras is the so-called Marra-Spada duality. Such duality states that the full subcategory of finitely presented MV-algebras is equivalent to the opposite of the category of *rational polyhedra with $\mathbb{Z}$-maps*. Although the seminal idea of a deep connection between MV-algebras and polyhedra can be found in previous works by many different authors (one can see [4] and the references within), the duality is proved in [3].

This valuable (and fairly recent) result shared new light on MV-algebras, since it allowed a transfer of knowledge from logic to algebraic geometry and back. The duality works by mapping each *rational* polyhedron $P \subseteq [0,1]^n$ into the algebra obtained by restricting the functions of the free n-generated MV-algebra to P. It was only fair to start wondering on the consequences of restricting the elements of a free n-generated MV-algebra to *any* polyhedron, non necessarily a rational one. This was the content of [1], where the authors define *polyhedral MV-algebras* and prove, among other results, a duality between the full subcategory of polyhedral MV-algebras and the category of polyhedra and $\mathbb{Z}$-maps.

Another major line of research in the framework of MV-algebras stemmed out from a simple remark: the standard example of an MV-algebra – that is the unit interval $[0,1]$ endowed with suitable operations – is closed with respect to the product of real numbers. Thus, several researchers have investigated expansions of MV-algebras powerful enough to axiomatize this product. Many different notions have arisen, but we focus our attention on *DMV-algebras* and *Riesz MV-algebras*. Intuitively, they are MV-algebras endowed with a scalar multiplication, where the scalars are taken in $[0,1] \cap \mathbb{Q}$ in the first case and in $[0,1]$ in the second. Building on the fact that suitable versions of the Marra-Spada duality are proved for both categories of finitely presented DMV-algebras and Riesz MV-algebras, we will see how it is possible to generalize the notion of polyhedral MV-algebras to the setting of DMV-algebras and to explore the connections with finitely presented MV-algebras, DMV-algebras and Riesz MV-algebras. In more details, we will present the following results:

(1) We will define polyhedral DMV-algebras ans we will use the hull-functors (defined via tensor product) to connect the various classes of polyhedral

algebras (polyhedral DMV-algebras, poyhedral MV-algebras, finitely presented Riesz MV-algebras).

(2) We will prove dualities with appropriate classes of polyhedra and we prove equivalences among some of the categories of algebras aforementioned.

(3) We will define several hom-functors and we will analyze, by tensor product, the interrelations of MV-algebras, DMV-algebras and Riesz MV-algebras, both in the case of polyhedral algebras and of finitely presented algebras.

(4) We will prove the amalgamation property for finitely presented DMV-algebras and Riesz MV-algebras and for polyhedral DMV-algebras.

The results presented are included in [2].

References

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Department of Mathematics, University of Salerno, Via Giovanni Paolo II, 132 Fisciano (SA), ITALY

Email address: slapenta@unisa.it