The Decomposition of Linearly Ordered Pseudo-Hoops

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1 Extended Abstract

Pseudo-hoops are partially ordered residuated monoids, not necessarily commutative, with the order given by the left- and right-divisibility, and they are also meet-semilattices, satisfying a divisibility condition. The structure was initially introduced and studied by Bosbach under the name of *hoops*, and no assumption on the commutativity of the monoid operation was made. However, subsequent studies focused on commutative hoops, until the non-commutative structure was again put in circulation by Georgescu, Leuştean and Preoteasa under the name *pseudo-hoops*.

Pseudo-hoops are weak structures. By adding axioms to them, one can obtain the algebras of non-commutative fuzzy logic: the pseudo-BL algebras, the pseudo-Wajsberg or the pseudo-MV algebras, the pseudo-product algebras. Pseudo-hoops are the algebraic counterpart of falsehood-free fragments of non-commutative fuzzy logics.

Mostert ans Shields proved that every continuous t-norm on $[0, 1]$ is locally (i.e. on specific open sub-intervals) isomorphic to: either Lukasiewicz's t-norm ($x \odot y = sup(0, x+y-1)$), or to the product t-norm ($x \odot y = xy$ product of reals), and, in between different components, the t-norm is Gödel ($x \odot y = min\{x, y\}$).

For linear commutative hoops several decompositions as ordinal sums were obtained. Hájek has generalized this result for linear BL-algebras, obtaining a decomposition into Wajsberg and product algebras. Cignoli, Esteva, Godo and Torrens continue the work of Hájek and obtain a decomposition with Gödel, product and Wajsberg components. Using a completely different construction, Agliano and Montagna obtain a decomposition of commutative hoops with only Wajsberg components. The constructions of Hájek and of Agliano and Montagna have been generalized to the non-commutative case by Dvurecenskij.

We generalize to the non-commutative case a construction by Laskowski and Shashoua, which use an equivalence relation to obtain the Hájek decomposition.

Using equivalence classes, we obtain not only the Hájek decomposition, but also the Agliano-Montagna one, and the Cignoli-Esteva-Godo-Torrens one, thus establishing also the comparability of the three decompositions.