Automorphisms in the n-generated free algebra in the subvariety of BL-algebras generated by $[0,1]_{\mathbf{MV}} \oplus [0,1]_{\mathbf{G}}$

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BL-algebras were introduced by Hájek (see [4]) to formalize fuzzy logics in which the conjunction is interpreted by continuous t-norms over the real interval $[0,1]$. These algebras form a variety, usually called $\mathcal{BL}$. In this work we will concentrate in the subvariety $\mathcal{MG} \subseteq \mathcal{BL}$ generated by the ordinal sum of the algebra $[0,1]_{\mathbf{MV}}$ and the Gödel hoop $[0,1]_{\mathbf{G}}$, that is, generated by $\mathbf{A} = [0,1]_{\mathbf{MV}} \oplus [0,1]_{\mathbf{G}}$. Though it is well-known that $[0,1]_{\mathbf{G}}$ is decomposable as an infinite ordinal sum of two-elements Boolean algebra, the idea is to treat it as a whole block. The elements of this block are the dense elements of the generating chain and the elements in $[0,1]_{\mathbf{MV}}$ are usually called regular elements of $\mathbf{A}$. The main advantage of this approach, is that unlike the work done in [3] and [1], when the number n of generators of the free algebra increase the generating chain remains fixed. This provides a clear insight of the role of the two main blocks of the generating chain in the description of the functions in the free algebra: the role of the regular elements and the role of the dense elements.

We have a functional representation for the free algebra $Free_{\mathcal{MG}}(n)$. To define this functions we need to decompose the domain $\mathbf{A}^n = ([0,1]_{\mathbf{MV}} \oplus [0,1]_{\mathbf{G}})^n$ in a finite number of pieces. In each piece a function $F \in Free_{\mathcal{MG}}(n)$ coincides either with McNaughton functions or functions of $Free_{\mathcal{G}}(n)$.

We study the automorphisms in $Free_{\mathcal{MG}}(n)$. The automorphisms are important since they help us to understand the symmetries of the logic. In the case of the algebra $Free_{\mathcal{MV}}(n)$, the automorphisms were studied, for example, in [5], and for the case of the algebra $Free_{\mathcal{G}}(n)$, we give a description based on the one given in [2] for finite Gödel algebras. Using both descriptions, we will show a characterization of automorphisms on $Free_{\mathcal{MG}}(n)$. We show that they coincide with automorphisms of $Free_{\mathcal{MV}}(n)$ on $([0,1]_{\mathbf{MV}})^n$, automorphisms of $Free_{\mathcal{G}}(n)$ on $([0,1]_{\mathbf{G}})^n$, and for the rest of the domain, they coincide with automorphisms of $Free_{\mathcal{G}}(m)$, for some $m < n$, on cylindrifications of simplexes of a unimodular triangulation of $([0,1]_{\mathbf{MV}})^n$.

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