Characterization by triples of varieties of bounded residuated lattices with an MV retract

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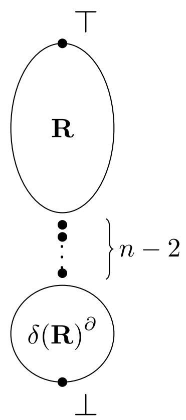
Residuated lattices constitute the algebraic semantics of substructural logics, which encompass many of the interesting nonclassical logics: intuitionistic logic, fuzzy logics, relevance logics, linear logic, besides including classical logic as a limit case. The investigation of the variety of residuated lattices is a powerful tool for analysing such logics uniformly, as deeply explored in [4], but the multitude of different structures makes the study fairly complicated, thus the structural investigation of interesting subvarieties is an appealing problem to address.

We introduce a new way of constructing bounded residuated lattices that generalizes the notion of disconnected and connected rotations, see [8], [5] and [6]. Given a residuated lattice $\mathbf{R}$ with an extra operation δ and the MV-chain Ł_n, we construct a bounded residuated lattice $\mathfrak{R}^{\delta}_{\mathbf{n}}(\mathbf{R})$, with lattice structure as follows.



We study the variety $\mathsf{MVR_n}$ generated by these generalized rotations and we show that it encompasses many different classes of bounded residuated lattices that were previously studied separately, including Stonean residuated lattices (for $n = 2$), regular Nelson lattices and in particular NM-algebras (for $n = 3$), and the variety of BL_n-algebras. While disconnected rotations of residuated structures generate varieties of algebras having a Boolean retraction term [3], our more general definition of rotation results in generating structures with a retraction term into an n-contractive MV-algebra. This retraction is the key to characterize these algebras in terms of suitably defined triples, made of an n-contractive MV-algebra, a residuated lattice with a nucleus, and an operator joining them.

The idea of representing algebras with a lattice underlying structure by means of triples started with Chen and Grätzers decomposition theorem for Stone algebras. More recently, triples decompositions have been studied for residuated algebraic structures. For example, in [7, 1] triples decompositions are introduced for a large class of MTL-algebras, while in [2] the authors use triples to characterize Stonean residuated lattices.

In this talk we will present the construction of generalized rotations starting from a residuated lattice with a nucleus and the MV-chain $\mathbf{Ł}_n$, and then proceed to describe the variety $\mathsf{MVR_n}$ generated by them - for a fixed n. Finally we will show how these can be represented by a suitably defined category, where the objects are triples consisting in an n-contractive MV-algebra $\mathbf{M}$, a residuated lattice $\mathbf{R}$ with a nucleus δ, and a bounded lattice morphism from the Boolean skeleton of $\mathbf{M}$ into the lattice of implicative filters of $\mathbf{R}$, obtaining a categorical equivalence.

References

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