

On some properties of the lattice of varieties of MTL-algebras

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Extended Abstract

Monoidal t -norm based logic MTL was introduced by Esteva and Godo in [5], and in [7] it is shown that MTL the logic of all left-continuous t -norms and their residua. The variety of MTL-algebras \mathbb{MTL} is the semantics of MTL. MTL and its axiomatic extensions are all algebraizable in the sense of Blok and Pigozzi [4], and during the years the a number of family of varieties of MTL-algebras has been extensively studied.

How about the general structure of the lattice of non-trivial varieties of MTL-algebras (ordered by inclusion), $\mathcal{L}_{\mathbb{MTL}}$? It is known that $\mathcal{L}_{\mathbb{MTL}}$ is a dually algebraic lattice, having the variety of Boolean algebras \mathbb{B} as bottom and \mathbb{MTL} as top: however, the structure of $\mathcal{L}_{\mathbb{MTL}}$ remains mostly unknown.

The recent papers [1, 3, 2] aimed to partially fill this gap, by analyzing some properties of $\mathcal{L}_{\mathbb{MTL}}$. In particular,

- [1] focuses on the study of almost minimal varieties of MTL-algebras, i.e. the atoms of $\mathcal{L}_{\mathbb{MTL}}$ (such notion was firstly introduced in [6, 8]).
- [3] is devoted to the so called strictly join irreducible varieties of MTL-algebras, which are the strictly join irreducible elements of $\mathcal{L}_{\mathbb{MTL}}$. Every element of $\mathcal{L}_{\mathbb{MTL}}$ (variety of MTL-algebras) is obtained as a join of strictly join irreducible varieties of MTL-algebras.
- [2] presents the notion of linear variety of MTL-algebras, i.e. a variety whose lattice of non-trivial subvarieties is a chain (an example is given by the variety \mathbb{G} of Gödel-algebras).

The purpose of this talk is - time permitting - to present such topics, possibly with new results related to these research lines.

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