Boolean Connexive Logics

Abstract

Tomasz Jarmużek & Jacek Malinowski

In this paper we define a new type of connexive logics which we call Boolean connexive logics. The Boolean operations: negation, conjunction and disjunction behave in a classical, Boolean way in such logics. We determine these logics through application of the relating semantics. In the final section of the paper, we present a tableau approach to the discussed logics.

The connexive logic is based on the theses set forth by Aristotle and Boethius, which only use negation and implication connectives. What is more, these theses are contradictive to the classical logic. Therefore, in the connexive logic we must interpret at least one of these connectives in a non-classical manner.

(A1) $\sim (A \Rightarrow \sim A)$

(A2) $\sim (\sim A \Rightarrow A)$

(B1) $(A \Rightarrow B) \Rightarrow \sim (A \Rightarrow \sim B)$

(B2) $(A \Rightarrow \sim B) \Rightarrow \sim (A \Rightarrow B)$.

In this study we shall only consider such connexive logics where the negation, conjunction and alternative have equal meanings with those in the classical logic. Thus each logic of this type we shall refer to as *Boolean connexive logic* since they preserve the meanings of the basic Boolean connectives.

The study offers a new approach to the issue of connexivity. Rather than using for instance the semantics of possible worlds or ternary accessibility relation — as the starting basis for the definition of the connexive logic — we shall assume a certain type of intensional logic: relating logic. By combing the semantic structures for relating logics with a Boolean language we obtain several different logics. The strongest ones among them include Aristotle's and Boethius' connexive laws as their tautologies. Hence, they are connexive logics.

Further in the study we present the following issues. First, we bring back some basis issues involved in the connexive logics. Further, we present the semantics of the relating logic which we shall uses as grounds for specification of our Boolean connexive logics systems and related issues. By dint of the findings concerning relations between the Aristotle's and Boethius' theses and the conditions imposed on the relating relation, we can present a lattice of logics comprising the least Boolean connexive logic along with a natural extension. Lastly, as a decision-making procedure, we propose the tableau methods that we shall elaborate in the last section of the study.