A non-linear, stochastic model for soil moisture

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Hydrologic models may solve the water cycle at different temporal and spatial resolutions, may be based on physical equations that describe the behaviour of the intervening processes or be based on conceptual equations that focus on capturing the main characteristics of the processes on a simplified manner. However, a common characteristic to them all is that they are effective models, that work with parameters that approximate the main characteristics of the terrain (difussion, permeability, capilarity, etc.). These models based on effective formulations may be simple to build and easy to use, and even constitute a reasonable approximation to some applications, but they lack predictive value when predictions are needed for conditions well outside the historical record. This problem is especially relevant to analyse the impact of climate change over the hydrologic cycle.

In this work, we develop a theoretical model that describes the behaviour of soil moisture using techniques inspired in the field of statistical physics. These mathematical techniques include the definition of fields that vary in space and time, stochastic partial differential equations that describe the behaviour of these fields, the description of the variable through probability functions and the computation of correlations among variables.

Previous work in this field has been presented in Chen et al. 2017, Rodriguez-Iturbe et al. 2006 and Isham et al. 2005. The model that we propose extends these previous works to include different non-linear components (saturation mechanism and disordered properties of the medium), expanding the range of applicability of the model beyond arid and semi-arid climates, and allowing to incorporate more sources of uncertainty into the analysis. We compare the predictions of our model and the previously cited models against field observations to show the improvement of our approach over previous ones.

The approach that we present serves as a starting point to build a fully stochastic hydrological model that considers all hydrologic fluxes as stochastic processes, incorporating the uncertainty in the properties of the medium in a natural way. The approach may also allow to incorporate the feedback between vegetation and hydrology in a more natural way, and thus be used to analyse the impacts of climate change beyond the water balance.

References

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