Efficient Branch-Current-Based Distribution System
State Estimator including Synchronized Measurements

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I. INTRODUCTION

State Estimation (SE) is generally used to determine the quantities of interest at unmónitored locations of a considered system. SE is based on mathematical relations between system state variables and available measurements. SE techniques are broadly exploited as support to make secure control operations in power systems. However, in the actual evolving scenario, traditional SE techniques could be insufficient to accurately evaluate the status of the network. Focusing on the last generation of distribution networks, the so called active networks, with increasing presence of Dispersed Generation (DG), storage devices and flexible loads to be controlled, new distributed and reliable measurement systems, provided with modern measurement devices, are required, so that the knowledge of the status of the network could be sufficiently accurate and evaluated with the needed reporting rate. In particular, the Phasor Measurement Units (PMUs), that are becoming increasingly widespread in transmission systems [1]–[5], are expected to be widely used also in distribution systems. The utility industry across the world is trying to modernize the distribution networks, considering, anyway, the best costs/benefits ratio. For this reason, the measurement devices will not be widespread in such systems. As a consequence, it would be still necessary to develop estimators specific for distribution systems, Distribution System State Estimators (DSSEs). Examples of DSSEs can be found in [6]–[14]. In these systems, indeed, for both historical and economic reasons, measurement devices actually present in the field are very limited with respect to transmission systems, so that, sometimes, measurements exist only in the substation. The estimation quality of the quantities of interest obviously reflects measurement system configuration. However, it has become essential to improve the accuracy of the estimation results, while enhancing measurement responsiveness and frequency. Thus, innovative solutions specific for the distribution systems of the future are increasingly needed. In particular, the diffusion of DG could create unforeseen dynamics, that require estimations obtained with the necessary accuracy and updating rate. In this context, PMUs appear very interesting, because they are able to accurate measure phasors of voltage and current, synchronized with the Coordinated Universal Time (UTC), supplying frequency and rate of change of frequency ([11], [15]). Furthermore, a significant reporting rate is selectable. In the area of transmission systems, PMUs have been included in mixed state estimators that rely both on SCADA and synchronized measurements, with different approaches [2], [3]. On the other hand, it is important to note that there are no satisfactory proposals for estimators that make efficient use of PMUs in distribution systems.

In literature, methodologies based on branch currents and considering the peculiar characteristics of distribution systems, as the radial or weakly meshed topology and the high \( r/x \) ratios, and efficient in treating current measurements have been proposed in [9]–[14]. Such estimators are faster with respect to those based on voltage state. In the seminal paper [9] a three-phase branch current DSSE was introduced using real and imaginary parts of branch currents as state variables. In [10] a fast and decoupled version of the BC-DSSE was presented. Both methods neglect voltage measurements. In [12], [13] voltage amplitude measurements were considered in the estimator. In [14] the BC-DSSE was designed using current amplitude and phase as state variables, allowing an easier employment of current amplitude measurements. In the
last years, the authors have been working on the problem of how to improve the accuracy in state estimation and harmonic state estimation procedures, see for example, [16]–[19]. In particular, in a previous paper [20], the authors proposed a method to include PMU measurements in a branch currents estimator (which basis can be found in [9], [13]). Besides, in [20] the model state was extended to include the slack-bus voltage, allowing to significantly improve the knowledge of the whole voltage profile.

In this paper, an improved version of the estimator presented in [20] is proposed. The formulation in both polar and rectangular coordinates (see Section III and Appendices A and B) is presented and discussed. The possibility to treat also weakly meshed topology, also in presence of dispersed generation has been introduced (see in particular Section III-C). Furthermore, three-phase version has been implemented and tested. The efficiency of the proposed method, a comparison between the accuracy and computational efficiency obtainable with polar or rectangular coordinates, and the impact on the estimation accuracy of the synchronized phasor measurements are presented and discussed on the basis of results obtained for different distribution networks.

II. BRANCH CURRENTS DISTRIBUTION SYSTEM STATE ESTIMATOR

The distribution networks have very few measurement devices available on the systems. As a consequence, knowledge obtained from a priori information has to be added to the real-time measurements to make the system observable. This prior information is commonly referred to as “pseudo-measurements” in power systems literature (see for example, [7], [8], [10], [14]). Several DSSESs, mainly based on WLS approach, have been developed in the last years, for the estimation of the node voltages or the branch currents. The state variables used for branch currents estimation can be both the currents amplitude and phase angles (as in [14]) or their real and imaginary parts ([9]).

In state estimation the general measurement model can be represented as:

\[ \mathbf{y} = \mathbf{h}(\mathbf{x}) + \mathbf{e} \]  

where \( \mathbf{y} = [y_1 \ldots y_M]^T \) is the vector of the measurements gathered from the network and of the chosen pseudo-measurements; \( \mathbf{h} = [h_1 \ldots h_M]^T \) is, in general, the vector of non-linear measurement functions (depending on the type of measurement); \( \mathbf{x} = [x_1 \ldots x_N]^T \) is the vector of the state variables. The measurement noise vector \( \mathbf{e} \) is often assumed to be composed by independent zero mean Gaussian variables, with covariance matrix \( \mathbf{\Sigma}_e = \text{diag}[\sigma^2_{e_1}, \ldots, \sigma^2_{e_M}] \).

The state \( \mathbf{x} \) is \( [i_1^r \ldots i_{N_b}^r, i_1^i \ldots i_{N_b}^i]^T \) if it is expressed in terms of the real \( i^r \) and imaginary \( i^i \) parts of the \( j \)-th branch current (\( N_b \) is the number of branches), whereas \( \mathbf{x} = [i_1 \ldots i_{N_b}, \theta_1 \ldots \theta_{N_b}]^T \) if a polar representation is chosen.

The measurements can be of several types, voltage, current, real and reactive power flows or power injections, while the pseudo-measurements are above all obtained from historical information on the powers drawn by the loads. Measurements have low standard deviation \( \sigma \), while the a priori values are assigned with a higher \( \sigma \) to highlight the lower confidence given to such quantities (quite low accuracy, due to the fact that their accuracy is based on non-measured data).

In the BC-DSSE approach the measurement model is often simplified by the definition of equivalent measurements. The estimation of the state \( \mathbf{x} \) in the BC-DSSE is obtained by an iterative three steps algorithm. Each iteration consists of:

- Definition/update of measurements and residuals;
- Branch currents estimation applying a Weighted Least Squares (WLS) method;
- Network voltages state computation through a forward sweep calculation, descending the network graph starting from the slack bus voltage.

In the first step, the measurement residuals are computed. For the first iteration, when estimates are not available, an initialization for the state variables is needed. In this paper, the same initial values as in [10] are used for branch currents. If equivalent measurements are employed, they have to be computed before calculating residuals. In particular, power measurements depend on both node voltages and injected or flowing currents. In [9] and [10] for instance, equivalent current measurements are defined, starting from the knowledge of the voltage state at iteration \( n - 1 \), that allow a straightforward integration in the estimator. In [14] instead, active and reactive power measurements model is kept non-linear.

The WLS step is performed minimizing the weighted sum of the squares of the residuals. At each iteration the state variables variation \( \Delta \mathbf{x} = \mathbf{x}_n - \mathbf{x}_{n-1} \) is computed starting from the measurement residuals, by solving the following equations:

\[ \mathbf{H}^T \mathbf{W} \Delta \mathbf{x} = \mathbf{H}^T \mathbf{W} (\mathbf{y} - \mathbf{h}(\mathbf{x}_{n-1})) \]  

where \( \mathbf{H} \) is the Jacobian of the measurement functions and \( \mathbf{W} \) is the weighting matrix, that is usually chosen equals to the inverse of \( \mathbf{\Sigma}_e \). \( \mathbf{H}^T \mathbf{W} \) represents the so called gain matrix. In Appendix A detailed expressions for the elements of the Jacobian, related to traditional measurements are reported for sake of completeness. The forward sweep step is performed to compute network voltages for each node, by a simple evaluation of voltage falls along the lines, given the branch currents estimated in the WLS.

III. THE PROPOSED METHOD

The main goal of the paper is to present a complete BC-DSSE providing accurate estimation also in case of active network, where, in particular, the quality of the estimates can seriously affect the system management. The procedure is designed to use the real-time measurements, obtained from the field, and all the available pseudo-measurements. According to [20], the slack bus voltage is added to the branch currents state to improve the knowledge of the voltage at the substation. This knowledge is fundamental because such voltage influences the whole voltage profile and can be influenced by unforeseen dynamics. In [9] it was highlighted that, if the value considered for the voltage at substation is not correct, the whole estimation is affected and, in particular, the estimation of the voltage state that can be derived by the forward sweep from the estimated currents. Adding new variables to the state vector impacts...
on the Jacobian $\mathbf{H}$ of the measurement functions, because the derivatives of each measurement with respect to the new variables have to be considered. In this paper both the polar and rectangular coordinates are used for state description, and the expressions of the Jacobian in presence of synchronized measurements and state extension are derived.

A. No PMU available in the network

When no phase measurement is available, phase angles of the network quantities have to be considered as phase differences from a common reference. Thus the slack bus is commonly chosen as the reference bus and only its voltage amplitude $v_{\text{slack}}$ can be included in the state and estimated.

If rectangular coordinates are used for branch currents (as in [20]), the new state vector for a network with $N_{\text{br}}$ branches can be chosen as:

$$\mathbf{x} = [v_{\text{slack}}, i_1^r, \ldots, i_{N_{\text{br}}}^r, i_1^i, \ldots, i_{N_{\text{br}}}^i]^T$$

With such state choice, the main differences in the expressions of the Jacobian in presence of synchronized and rectangular coordinates are used for state description, and variables have to be considered. In this paper both the polar and rectangular coordinates are used for branch currents (as in [20]), the new state vector for a network with $N_{\text{br}}$ branches can be chosen as:

$$\mathbf{x} = [v_{\text{slack}}, i_1^r, \ldots, i_{N_{\text{br}}}^r, i_1^i, \ldots, i_{N_{\text{br}}}^i]^T$$

With this state choice, the main differences in the Jacobian $\mathbf{H}$ appear in the rows corresponding to each voltage amplitude measurement $v_h$. As described in [13], at each iteration $n$:

$$v_h = |v_{\text{slack}} - \sum_{j \in \Gamma_h} z_j \cdot i_j|$$

$$\approx \text{Re} \left[ v_{\text{slack}} e^{-j\delta_{v_h}^{(n-1)}} - \sum_{j \in \Gamma_h} z_j i_j e^{-j\delta_{v_h}^{(n-1)}} \right]$$

$$= v_{\text{slack}} \cos(\delta_{v_h}^{(n-1)}) - \sum_{j \in \Gamma_h} (r_j i_j^r - x_j i_j^i) \cos(\delta_{v_h}^{(n-1)}) + \sum_{j \in \Gamma_h} (x_j i_j^r + r_j i_j^i) \sin(\delta_{v_h}^{(n-1)})$$

where $\delta_{v_h}^{(n-1)}$ is the phase angle of $v_h$ at the previous step, $z_j = r_j + jx_j$ and $i_j$ are $j$-th branch impedance and current respectively and $\Gamma_h$ represents the path from the slack bus to bus $h$. In the following, the iteration index will be suppressed for simplicity.

The new term of the Jacobian is then:

$$\frac{\partial v_h}{\partial v_{\text{slack}}} = \cos(\delta_{v_h})$$

leading to the following row for each voltage amplitude measurement:

$$v_h \rightarrow \left[ \cos(\delta_{v_h}) \quad \frac{\partial v_h}{\partial i_j^r} \quad \frac{\partial v_h}{\partial i_j^i} \right]$$

where (for a branch $j$ that belongs to the path $\Gamma_h$):

$$\frac{\partial v_h}{\partial i_j^r} = -r_j \cos(\delta_{v_h}) - x_j \sin(\delta_{v_h})$$

$$\frac{\partial v_h}{\partial i_j^i} = x_j \cos(\delta_{v_h}) - r_j \sin(\delta_{v_h})$$

When polar coordinates are used, (4) can be written as:

$$v_h = v_{\text{slack}} \cos(\delta_{v_h}) - \sum_{j \in \Gamma_h} z_j i_j \cos(\alpha_{z_j} + \theta_j - \delta_{v_h})$$

where $z_j$ and $\alpha_{z_j}$ are the branch impedance magnitude and phase. The corresponding row in the Jacobian becomes:

$$v_h \rightarrow \left[ \cos(\delta_{v_h}) \quad \frac{\partial v_h}{\partial i_j^r} \quad \frac{\partial v_h}{\partial i_j^i} \right]$$

From the above equations, it is clear that the voltage magnitude measurements add non constant terms in the measurement Jacobian, and thus the solution of (2) requires a new factorization of the gain matrix at each iteration.

B. At least one PMU available in the network

When at least one PMU is available, it is possible to estimate all the “absolute” phase angles values with respect to the Coordinated Universal Time (UTC) reference (in [21] the possibility to avoid a reference bus without drawbacks in the estimation is discussed for transmission systems).

In the BC-DSSS context, the state can be extended to include also the slack bus phase angle state. In particular, in the rectangular form, the state becomes:

$$\mathbf{x} = [v_{\text{slack}}^r, v_{\text{slack}}^i, i_1^r, \ldots, i_{N_{\text{br}}}^r, i_1^i, \ldots, i_{N_{\text{br}}}^i]^T$$

where $v_{\text{slack}}^r$ and $v_{\text{slack}}^i$ are the real and imaginary parts of the slack bus voltage respectively. It is important to notice that the slack bus voltage does not represent the reference for the phase angles anymore and is included in the state only to enhance estimation accuracy.

In case of traditional voltage magnitude measurements, the new Jacobian terms (equivalent of (5) in the new formulation) are:

$$\frac{\partial v_h}{\partial v_{\text{slack}}^r} = \cos(\delta_{v_h}) \quad \frac{\partial v_h}{\partial v_{\text{slack}}^i} = \sin(\delta_{v_h})$$

From the above terms it is clear that a magnitude voltage measurement impacts on the estimation of both the real and imaginary parts of the slack voltage.

In case of phasorial measurement performed by PMU, in the proposed estimator, both measured voltages and currents can be expressed in terms of their real and imaginary parts. Then, for each synchronized measure two rows have to be added to $\mathbf{H}$.

For PMU voltage measurement at node $h$, the added Jacobian rows are:

$$v_h^r \rightarrow \left[ \frac{\partial v_h^r}{\partial v_{\text{slack}}^r} \quad \frac{\partial v_h^r}{\partial v_{\text{slack}}^i} \quad \frac{\partial v_h^r}{\partial i_j^r} \quad \frac{\partial v_h^r}{\partial i_j^i} \right]$$

$$v_h^i \rightarrow \left[ \frac{\partial v_h^i}{\partial v_{\text{slack}}^r} \quad \frac{\partial v_h^i}{\partial v_{\text{slack}}^i} \quad \frac{\partial v_h^i}{\partial i_j^r} \quad \frac{\partial v_h^i}{\partial i_j^i} \right]$$

with $j = 1 \ldots N_{\text{br}}$. 

where:

$$\frac{\partial v_h}{\partial \delta_{v_h}^{(n-1)}} = -z_j \cos(\alpha_{z_j} + \theta_j - \delta_{v_h}^{(n-1)})$$

$$\frac{\partial v_h}{\partial \theta_j} = \pm \|\mathbf{H}_{jN_{\text{br}}}^{(n-1)}\|$$

From the above equations, it is clear that the voltage magnitude measurements add non constant terms in the measurement Jacobian, and thus the solution of (2) requires a new factorization of the gain matrix at each iteration.
Eqs. (17) clearly reflect the fact that the state currents components are directly measured. An expression analogous to (15) can be derived for current measurements, and similar considerations about the weighting matrix can be applied.

\[
\frac{\partial v_h^r}{\partial v_{slack}^r} = 0, \quad \frac{\partial v_h^r}{\partial v_{slack}^r} = 1 \quad (14a)
\]

\[
\frac{\partial i_j^r}{\partial v_{slack}} = 0, \quad \frac{\partial i_j^r}{\partial v_{slack}} = 1 \quad (14b)
\]

\[
\frac{\partial v_h^r}{\partial i_j^r} = -r_j, \quad \frac{\partial v_h^r}{\partial i_j^r} = x_j \quad (14c)
\]

\[
\frac{\partial v_h^r}{\partial i_j^r} = -x_j, \quad \frac{\partial v_h^r}{\partial i_j^r} = -r_j \quad (14d)
\]

where \(v_h^r\) and \(v_h^r\) are the real and imaginary parts of the voltage at node \(h\), and \(r_j\) and \(x_j\) are the real and imaginary parts of the impedance of \(j\)-th branch. When branch \(j\) is on the path from slack bus to the measured node the above equations are valid, otherwise the corresponding partial derivatives (14c) and (14d) are equal to zero. It is interesting to note that PMU voltage measurements have an impact on slack bus voltage estimation. It is clear from (14) that Jacobian terms are constant among the algorithm iterations, thus allowing a faster computation.

It is useful to recollect that, if the accuracy of PMU measurements is given in terms of magnitude and phase angle, the following expression, derived by direct application of [22], is used for the covariance matrix of \(v_h = [v_h^r v_h^\theta]^T\):

\[
\Sigma_v = \begin{bmatrix}
\cos(\delta_h) & -v_h \sin(\delta_h) \\
\sin(\delta_h) & v_h \cos(\delta_h)
\end{bmatrix} \times \begin{bmatrix}
\sigma_{\delta_h}^2 & 0 \\
0 & \sigma_{v_h}^2
\end{bmatrix} \times \begin{bmatrix}
\cos(\delta_h) & -v_h \sin(\delta_h) \\
\sin(\delta_h) & v_h \cos(\delta_h)
\end{bmatrix}^T
\]

where \(\sigma_{\delta_h}^2\) and \(\sigma_{v_h}^2\) are the covariances of amplitude and phase measurements respectively. In the WLS algorithm the weights of such equivalent PMU voltage measurements are chosen as the inverse of the \(2 \times 2\) covariance matrix, thus considering also their cross-correlation.

For PMU current measurement at branch \(h\), the added Jacobian rows are:

\[
i_h^r \rightarrow \begin{bmatrix}
\frac{\partial v_h^r}{\partial i_j^r} & \frac{\partial v_h^r}{\partial i_j^r} & \ldots & \frac{\partial v_h^r}{\partial i_j^r} & \frac{\partial v_h^r}{\partial i_j^r} \\
\frac{\partial v_h^r}{\partial i_j^r} & \frac{\partial v_h^r}{\partial i_j^r} & \ldots & \frac{\partial v_h^r}{\partial i_j^r} & \frac{\partial v_h^r}{\partial i_j^r}
\end{bmatrix}
\]

\[
i_h^r \rightarrow \begin{bmatrix}
\frac{\partial v_h^r}{\partial i_j^r} & \frac{\partial v_h^r}{\partial i_j^r} & \ldots & \frac{\partial v_h^r}{\partial i_j^r} & \frac{\partial v_h^r}{\partial i_j^r} \\
\frac{\partial v_h^r}{\partial i_j^r} & \frac{\partial v_h^r}{\partial i_j^r} & \ldots & \frac{\partial v_h^r}{\partial i_j^r} & \frac{\partial v_h^r}{\partial i_j^r}
\end{bmatrix}
\]

The following equations represent the terms of (16):

\[
\frac{\partial v_h^r}{\partial v_{slack}^r} = 0, \quad \frac{\partial v_h^r}{\partial v_{slack}^r} = 0 \quad (17a)
\]

\[
\frac{\partial v_h^r}{\partial v_{slack}^r} = 0, \quad \frac{\partial v_h^r}{\partial v_{slack}^r} = 0 \quad (17b)
\]

\[
\frac{\partial v_h^r}{\partial i_j^r} = \begin{cases} 1 \text{ when } j = h, \\ 0 \text{ elsewhere} \end{cases} \quad (17c)
\]

\[
\frac{\partial v_h^r}{\partial i_j^r} = \begin{cases} 1 \text{ when } j = h, \\ 0 \text{ elsewhere} \end{cases} \quad (17d)
\]

Eqs. (17) clearly reflect the fact that the state currents components are directly measured. An expression analogous to (15) can be derived for current measurements, and similar considerations about the weighting matrix can be applied.

If polar coordinates are used for the state, PMU current measurements can be straightforwardly included in the model, because they directly measure the state variable. In this case, the weights of the measured amplitude and phase angle are directly given by measurements variances. The slack bus state variables can be included in this case both in terms of polar and rectangular coordinates. When \(v_{slack}^r\) and \(v_{slack}^r\) are used the Jacobian terms that describe slack voltage influence on traditional voltage amplitude measurement are equal to those in (12).

Two options are also available for voltage phasor measurements that can be included both in their polar and rectangular form. However, when branch currents are expressed in polar form, it is not possible to find constant Jacobian terms, even for voltage PMU measurements in contrast to what happens when the state is in rectangular coordinates. In fact, the derivatives in (14c) and (14d) translate into:

\[
\frac{\partial v_h^r}{\partial i_j^r} = -z_j \sin(\alpha_{z_j} + \theta_j), \quad \frac{\partial v_h^r}{\partial i_j^r} = z_j \sin(\alpha_{z_j} + \theta_j) \quad (18a)
\]

\[
\frac{\partial v_h^r}{\partial i_j^r} = -z_j \cos(\alpha_{z_j} + \theta_j), \quad \frac{\partial v_h^r}{\partial i_j^r} = -z_j \cos(\alpha_{z_j} + \theta_j) \quad (18b)
\]

where the dependency of the expression from the current amplitude and phase angles is evident. For sake of clearness and to avoid an excess of equations and different cases, the terms that appear in the Jacobian when the slack voltage and voltage phasor measurements are treated in polar form are reported in details in Appendix B.

C. Weakly meshed networks

Distribution networks, in a Smart Grid scenario, could present some meshes in the topology. Each mesh adds a constraint on branch currents due to the correspondent Kirchhoff’s Voltage Law (KVL):

\[
\sum_{j \in A} \lambda_j z_j i_j = 0 \quad (19)
\]

where \(A\) is the set of branches belonging to the mesh and \(z_j\) and \(i_j\) are the impedance and current phasor of the \(j\)-th branch. \(\lambda_j\) can be +1 or −1 depending on the reference mesh direction with respect to branch \(j\) direction. In [9, 10] the KVLs of the meshes are added to the equation system (2) using Lagrange multipliers. In this paper, as usually done with zero injections, (19) is treated as a virtual measurement with high weight in the WLS. This approach, while giving same estimation accuracy results, allows to keep lower the system dimension.

IV. TESTS AND RESULTS

In order to analyze the performance of the proposed BC-DSSE, several simulations have been executed on different networks simulated by means of Matlab 2010 environment.

The following values are assumed:

- Number of Monte Carlo trials \(N_{MC} = 50000\);
- A maximum deviation of 50 % with respect to the nominal values for the active and reactive powers (Gaussian distribution) drawn by the loads;
Measurements assumed to be distributed as a Gaussian distribution with a standard deviation equal to a third of the accuracy value.

As for the PMUs, the following accuracy values are considered: 1% for the magnitude and 1 rad (10^-2 rad) for the phase angle; whereas for non-synchronized measurements, 1% for the magnitude of the voltage and 3% for the power flow and current magnitude measurements are assumed (as in [23], [24]). It is important to notice that quite a large uncertainty is assigned to synchronized measurement. The synchrophasors standard [15] allows a 1% vectorial error in phasor estimation, that can be translated into a maximum deviation of 1% in amplitude estimation or a maximum deviation of 1 rad for phase angle. Such limits are given for every test condition, except for the amplitude and phase modulations. In the last case a 3% vectorial error is allowed (see [15] for the details on test design). However, it has been shown (for instance in [25]) that there are algorithms designed for achieving phasorial measurements with much better compliance levels than required. On the contrary, quite low uncertainties are assumed for traditional measurements, almost neglecting the impact of the lack of synchronization (discussed in [19]). This approach is used to design a “worst-case” scenario in the comparison between PMUs and traditional measurements.

To assess the performance of the proposed procedure, for a given measurement configuration $N_{MC}$ possible operative conditions are considered. In each operative condition, the chosen measurements and active and reactive powers at the buses are extracted from their probability distributions. Reference values are evaluated for all the quantities of interest, starting from the nominal values of the system. Then, the estimation process is performed and the differences between the estimated and the reference values are computed for each possible operative condition. For the sake of simplicity several measurement configurations have been considered, without applying a meter placement technique. Measures have been located, as an example, in different points of common coupling of the network. In particular, two types of measurement points have been considered:

Type 1. “Traditional” measurement point: amplitude voltage measure along with branch current amplitude(s) measurements.

Type 2. “Synchronized” measurement point: amplitude and phase angle of both voltage and branch current(s) measured by PMU.

A. 18-bus U.K. radial feeder

Fig. 1 shows the considered small distribution network [26] that is a simplified version of a 18-bus U.K. radial feeder. The rated voltage is 11kV. Details on the network (line parameters and transformer characteristics) can be found in [26].

A considerable number of measurements are performed in this system. In the figure, both nodes and branches are numbered, the latter ones are indicated with a number in a square label. As an example of possible measurement scenarios, two test cases are presented:

Case 1. A measurement point in the node 6 is considered.

Case 2. Two measurement points in nodes 4 and 11 are considered.

In both cases, the substation voltage is considered to be monitored, and each measurement point is constituted by one node voltage and two current measurements (the currents of the branches downstream the node).

In the first series of tests, to better highlight the performance of the estimator, no DG was considered in the network.

As preliminary test, a comparison of the extended state model (11) and the traditional one is performed in Case 2 scenario. Table I shows the results in terms of Root Mean Square Error (RMSE) obtained with type 2 measurement points and considering state model expressed in rectangular coordinates. The minimum and maximum RMSEs (obtained averaging on $N_{MC}$) of the magnitude voltage estimations of the network nodes, calculated from the estimated branch currents, are reported. It is clear that a noteworthy error reduction (higher than 30%) is obtained by the new estimation paradigm. The importance of the voltage measurements in voltage profile evaluation appears evident.

<table>
<thead>
<tr>
<th>Table I</th>
<th>Minimum and maximum Root Mean Square Errors (RMSEs) including or not $v_{slack}^r$, $v_{slack}^i$ into state space model.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model Type</td>
<td>Without $v_{slack}^r$, $v_{slack}^i$</td>
</tr>
<tr>
<td>Min RMSE (%)</td>
<td>Max RMSE (%)</td>
</tr>
<tr>
<td>Without $v_{slack}^r$, $v_{slack}^i$</td>
<td>0.33</td>
</tr>
<tr>
<td>With $v_{slack}^r$, $v_{slack}^i$</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Results evaluated with the state in polar coordinates are practically identical and the consideration holds for the estimation error values of all the performed tests. The true difference between the two state models is in the efficiency and appears evident from Table II, where the average iteration number and execution time obtained considering Case 2 scenario are reported. These and following tests have been performed on an Intel Dual Core 3.17 GHz.

<table>
<thead>
<tr>
<th>Table II</th>
<th>Comparison of the computational efficiency of the state models in polar or rectangular form</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>Measurement</td>
</tr>
<tr>
<td>rectangular</td>
<td>type 1</td>
</tr>
<tr>
<td></td>
<td>type 2</td>
</tr>
<tr>
<td>polar*</td>
<td>type 1</td>
</tr>
<tr>
<td></td>
<td>type 2</td>
</tr>
</tbody>
</table>

* For a fair comparison of execution times, equivalent currents have also been used instead of power injection measurements.

Polar state is more efficient in treating the traditional amplitude current measurements (that become linear), however
the overall average execution time is affected by the heavier evaluation of Jacobian terms corresponding to injection pseudo-measurements. In the rectangular form, the estimation algorithm in presence of PMU measurements has a reduced computational cost because Jacobian matrix (and thus the gain matrix of WLS) can be designed to be constant. Since the estimation accuracies are equivalent, in the following, only the results obtained with the rectangular form will be presented.

A reduced computational cost of the estimator, along with the available high reporting rate of PMUs, are very useful for state estimation applications, above all considering the evolving scenario of distribution systems, where high dynamics is expected in the quantities of interest.

Focusing on the estimation results, a comparison between results obtained with type 1 and type 2 measurement points in Case 1 is shown in Figs. 2-3 and in Table III in terms of RMSEs. In particular, Fig. 2 shows the comparison of the percentage results obtained for the amplitude of the currents at every branch, while Fig. 3 shows the results for the phase angles (in radians). In all the figures the blue dashed line represents the type 1 results, whereas the continuous black line corresponds to type 2 results. It is possible to observe that even one PMU measurement point significantly impacts both on amplitude and phase angle of the branch currents estimation errors. Table III shows the results obtained for node voltages. The influence of the synchronized measurement in this case is not so evident. In this regard, it is important to recall that voltage magnitude measurements give the main contribution to the amplitude estimation and thus similar results were expected since, with the chosen conservative hypothesis, both traditional and synchronized voltage magnitude measurements have the same accuracy.

Fig. 4 reports the estimations of the branch current phase angles obtained in Case 2 configuration, with two measurement points. In this and in the following tests, also the results obtained using a mixed (traditional-synchronized) measurement system, with a traditional measurement in the node 11 and a synchronized one in the node 4, are shown. This hybrid system is indicated as “mixed” in the following tables and labelled as “mixed meas.” in the figures.

### TABLE III

<table>
<thead>
<tr>
<th>Estimation</th>
<th>Measurement</th>
<th>Min RMSE</th>
<th>Max RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>amplitude</td>
<td>type 1</td>
<td>0.23 [%]</td>
<td>0.25 [%]</td>
</tr>
<tr>
<td></td>
<td>type 2</td>
<td>0.24 [%]</td>
<td>0.24 [%]</td>
</tr>
<tr>
<td>phase angle</td>
<td>type 1</td>
<td>0.13 [rad]</td>
<td>0.14 [rad]</td>
</tr>
<tr>
<td></td>
<td>type 2</td>
<td>0.12 [rad]</td>
<td>0.13 [rad]</td>
</tr>
</tbody>
</table>

### TABLE IV

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Current amplitude RMSE</th>
<th>Voltage amplitude RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min (%)</td>
<td>Max (%)</td>
</tr>
<tr>
<td>type 1</td>
<td>0.71</td>
<td>14.70</td>
</tr>
<tr>
<td>mixed</td>
<td>0.29</td>
<td>14.70</td>
</tr>
<tr>
<td>type 2</td>
<td>0.24</td>
<td>14.70</td>
</tr>
</tbody>
</table>

The presence of DG on the network has been also investigated. Two generators on nodes 7 (0.25 MW and 75 kvar) and 14 (0.5 MW and 50 kvar) have been considered.

Fig. 5 shows the results in terms of RMSEs for the phase angles of the branch currents. The impact of DG is evident. In case of traditional measurements, a significant percentage error exists in the estimation of branch 10 current (adjacent to node 11, where a measurement point exists in Case 2 configuration): without information on the phases of currents 11 and 12, several pseudo-measurement configurations can match the measured current amplitudes independently of phase values, thus leading to significant errors in the low current 10
estimation in some of the \(N_{31C}\) trials. This fact is typical of distribution systems due to the small number of measurement points and the high uncertainty of prior information. The same kind of problem was highlighted in [27], where it is stated that the lack of measurements able to provide phase information can cause algorithm convergence to a local minimum having a reverse flow direction to the actual one or even convergence problems. By considering non-synchronized measurements, this problem can be partially faced using more branch current measurements or simultaneous active and reactive power flows measurements.

Table V summarizes the results obtained for node voltages. In this case, the influence of synchronized measurements is significant above all for phase angle estimation.

As further analysis, the impact of DG position was also investigated. The original network has been modified to test also DG presence in points of common coupling (PCC). All the carried out tests gave results similar to those in Fig. 5 and Table V. As an example, the results obtained for branch currents phase angles when DG is moved from node 7 to node 6 are showed in Fig. 6.

The possibility to have a weakly meshed topology has also been taken into account considering two closed tie switches, between nodes 18 - 8, and 18 - 14 (dashed lines in Fig. 1).

Fig. 7 shows the results in terms of RMSEs for the phase angle of the branch currents. Again it is possible to observe the impact of synchronised measurements on the improvement of estimation quality. As for the branch current amplitude estimation, in this case, the accuracy slightly worsens in some branches, as reported in Table VI. It is worth noting that, in any cases, a higher number of measurement devices would be required in order to obtain an accurate knowledge of the status of the branch currents in the network. As for the node voltages, also in this case, the impact of synchronized measurements on the voltage phase angle RMSEs is evident, with estimation results equivalent to those reported in Table V.

It is interesting to highlight that including meshes by means of virtual measurements (see Section III-C) leads to a fastest computation (more than 10% in the tests): the number of iterations is the same but the system is smaller and the degree of sparsity in gain matrix increases.

An example of application of the proposed estimator to a larger network is also presented.

**B. 95-bus U.K.**

Fig. 8 shows a part of the U.K. Generic Distribution Network (UKGDS) model. The considered system comprises 95 buses and 94 branches, with two sources of DG \(^1\). The network and load data for UKGDS were obtained from [23], [28].

As example of the results obtained for this network Figs. 9 and 10 can be considered. These results present the RMSEs for the estimation of branch currents and node voltage phase angles respectively, in the measurement scenario 3 of [29], with current measurements instead of power flow measurements. An hybrid measurement configuration has also been

\(^1\)Numeration of the branches follows the same criterion used for 18 bus network: each branch index is given by the node number of its end node (the larger one), decreased by one.
considered by using two type 1 measurements (on nodes 26 and 27) and two type 2 measurements (on nodes 1 and 25).

![Fig. 8. Test system 95-bus](image)

![Fig. 9. 95-bus. RMSE of branch current phase angle estimations](image)

![Fig. 10. 95-bus. RMSE of node voltage phase angle estimations](image)

Also in this situation, synchronized measurements have proven to guarantee a significantly more accurate estimation of the network status. RMSEs for branch current phase angles are more than halved and dramatically reduced as for node voltage phase angles estimation. Table VII reports the impact of the network status. RMSEs for branch current phase angles estimation. Table VIII shows iteration number and execution times for the considered test. It is worth noting that, in case of mixed measurements, it is not possible to define a constant gain matrix. In comparison with type 1 measurements results, the higher number of state variables and measurements leads to a larger equation system thus determining, despite of a smaller iteration number, higher execution times.

C. 13-bus unbalanced network IEEE

The estimator has also been tested with IEEE 13 nodes unbalanced network [30]. In order to deal with unbalanced systems, a three-phase line model of the network, taking into account the magnetic coupling between the phases, has to be used (see for example [9]). Thus, for each branch, a 3x3 impedance matrix \( Z \) is obtained, where \( z_{ij} \) terms represent the self-impedances and \( z_{ij} \) the mutual ones. Given the asymmetry of the electrical quantities, the state vector has to be expressed, with analogy to (11), in the three-phase version:

\[
x = [v_{\text{slack},A}^r, v_{\text{slack},A}^i, v_{\text{slack},B}^r, v_{\text{slack},B}^i, v_{\text{slack},C}^r, v_{\text{slack},C}^i, i_{N_{\text{br}},A}^r, i_{N_{\text{br}},A}^i, i_{N_{\text{br}},B}^r, i_{N_{\text{br}},B}^i, i_{N_{\text{br}},C}^r, i_{N_{\text{br}},C}^i]^T
\]  

(20)

In the three phase formulation, all equations in Section III still hold, but analogue derivatives have to be calculated also with respect to the other phases. Mutual impedances determine coupling among the different phases for both traditional and synchronized voltage measurements. In fact, mutual terms \( r_{ij} \) and \( x_{ij} \) (real and imaginary part of \( z_{ij} \), respectively) appear in the derivatives of each voltage measurements with respect to the currents of the other phases. It is worth noting that self-impedances are generally greater than mutual terms and, as pointed out also in [10], BC-DSSE allows to obtain a decoupled version of the WLS step by neglecting mutual impedances. Then, in this step, the state of each phase can be considered separately and the WLS can be performed by solving three smaller equation systems. However, in both coupled and decoupled version, mutual terms are considered in the forward sweep step and thus no approximation exists in the network modeling.

Tests have been performed to assess the proposed estimator both in coupled and decoupled versions and confirm that phase-decoupling is possible in presence of both traditional and synchronized measurements. Fully coupled estimator gives slightly better accuracy (as reported in Table IX) but lower execution times are obviously obtained with the decoupled version since decoupled WLS is faster.

D. Discussion on real time applications

In previous subsections, a discussion on execution times obtained by simulation in a PC Matlab environment was
reported. The aim was to highlight the performance of the algorithms and different behaviours. It is worth noting that specific time and implementation requirements strongly depend on the application and Distribution System Operator policies and priorities. Research activities aimed at deployment of applications for distribution systems are in progress. Examples of implementations using industrial PCs both in Matlab and other environments can be found in [31], [32]. Applications are many and evolving very fast (above all considering the smart grid paradigm) and thus their time cycles can vary on a wide range. The final application working cycle depends, not only on the state estimation module, but on the whole chain (measurement, communication, estimation, decision and control). PMUs can reach really high reporting rates, 50/s, nevertheless, considering latency (and, in particular, its jitter) and data concentration architecture, reporting rate of about 10/s will be a more realistic objective. For example in [33] the state estimation is expected to be performed multiple times per second. In this regard, it is worth noting that DSSE speed is related above all to the size of the network. As a consequence, possibility of network splitting, node aggregation and parallelization can be sought.

V. CONCLUSION

In this paper an efficient state estimation algorithm aimed at estimating the status of a distribution system is presented. The estimator is developed to use both traditional, non-synchronized measurements, and synchronized ones, obtained from PMUs. Furthermore, the state model is extended to include the slack-bus voltage in the estimation process, so that the knowledge of the whole voltage profile can be significantly improved. The estimator can be expressed in polar or rectangular coordinates. Furthermore, the possibility to treat both radial and weakly meshed topology, also in presence of DG, is shown. Test results obtained on three distribution networks, two balanced and one unbalanced, are presented and discussed to highlight the efficiency of the proposed procedure. The impact of PMUs usage on the estimation accuracy is investigated. Results prove that the state expressed in rectangular form is computationally more efficient, unless a large number of current measurements exists. In fact, in presence of PMU measurements, it is possible to define a constant gain matrix. All the performed tests, considering different measurement configurations, topologies and DG location, show that PMUs measurements can significantly impact on the accuracy of estimations, in particular for networks with DG or with weakly meshed topology.

TABLE IX
IEEE 13-bus. Average RMSEs results.

<table>
<thead>
<tr>
<th>Meas.</th>
<th>Version</th>
<th>Current magnitude [%]</th>
<th>Current phase [rad]</th>
<th>Voltage magnitude [%]</th>
<th>Voltage phase [rad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>type 1</td>
<td>coupled</td>
<td>9.16 8.25</td>
<td>0.26</td>
<td>0.18</td>
<td></td>
</tr>
<tr>
<td></td>
<td>decoupled</td>
<td>9.17 8.29</td>
<td>0.26</td>
<td>0.19</td>
<td></td>
</tr>
<tr>
<td>type 2</td>
<td>coupled</td>
<td>8.98 6.91</td>
<td>0.25</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td></td>
<td>decoupled</td>
<td>8.99 6.91</td>
<td>0.25</td>
<td>0.12</td>
<td></td>
</tr>
</tbody>
</table>

APPENDIX A

In a rectangular coordinates framework, power measurements are translated into equivalent current measurements (see [10]):

$$i_{eq} = \frac{(P + jQ)\ast}{v_e} = i_{eq}^r + j i_{eq}^i$$  \hspace{1cm} (A.21)

where $P$ and $Q$ are respectively real and reactive powers and $v_e$ is the node voltage estimated at previous iteration. Equivalent current measurements, that can represent flowing or injected currents, are treated as the current phasorial measurement (see Section III) and the corresponding non-zero elements in the Jacobian are +1 and −1, when the branch current contributes to the given measurement.

Current amplitude measurement at branch $j$ contribute to the Jacobian with the following derivatives:

$$\frac{\partial i_j}{\partial i_{ij}} = \cos(\theta_j) \quad \frac{\partial i_j}{\partial i_{ij}} = \sin(\theta_j) \hspace{1cm} (A.22)$$

When polar coordinates are used, as in [14], power flow measurements of branch $j$ (measured at node $h$) are expressed as:

$$P_j = v_h i_j \cos(\delta_{vh} - \theta_j) \quad Q_j = v_h i_j \sin(\delta_{vh} - \theta_j) \hspace{1cm} (A.23)$$

where $v_h$ and $\delta_{vh}$ are respectively magnitude and phase angle of node $h$ estimated at previous iteration. Thus the terms of the Jacobian corresponding to the real and reactive powers are:

$$\frac{\partial P_j}{\partial i_j} = v_h \cos(\delta_{vh} - \theta_j) \quad \frac{\partial P_j}{\partial \theta_j} = -v_h i_j \sin(\delta_{vh} - \theta_j) \hspace{1cm} (A.24a)$$

$$\frac{\partial Q_j}{\partial i_j} = v_h \sin(\delta_{vh} - \theta_j) \quad \frac{\partial Q_j}{\partial \theta_j} = v_h i_j \cos(\delta_{vh} - \theta_j) \hspace{1cm} (A.24b)$$

Power injection measurements generate analogue terms for each branch connected to the given bus. Only the sign of the above terms can change if the branch is an ingoing branch.

Each branch current amplitude measurement is easily inserted because the state already contains the corresponding variable.

APPENDIX B

When polar coordinates $(v_{slack}, \delta_{v_{slack}})$ of the slack bus voltage are added to the polar branch current state the new state becomes:

$$x = [v_{slack}, \delta_{v_{slack}}, i_1 \ldots i_N, \theta_1 \ldots \theta_N]^{T} \hspace{1cm} (B.25)$$

The row of the Jacobian corresponding to voltage amplitude measurements thus includes (10) and the new terms (equivalent of (12)):

$$\frac{\partial \delta_{vh}}{\partial v_{slack}} = \cos(\delta_{v_{slack}} - \delta_{vh}) \quad \frac{\partial \delta_{vh}}{\partial \theta_{slack}} = -\sin(\delta_{v_{slack}} - \delta_{vh}) \hspace{1cm} (B.26)$$

Voltage phasor measurements can be treated both in their rectangular and polar form. In the first case, the derivatives
with respect to the slack bus are:

\[
\frac{\partial v_h^*}{\partial \delta_{\text{slack}}} = \cos(\delta_{\text{slack}}) \quad \frac{\partial v_h^*}{\partial v_{\text{slack}}} = -v_{\text{slack}} \sin(\delta_{\text{slack}}) \quad \text{(B.27a)}
\]

\[
\frac{\partial v_h^*}{\partial v_{\text{slack}}} = \sin(\delta_{\text{slack}}) \quad \frac{\partial v_h^*}{\partial \delta_{\text{slack}}} = v_{\text{slack}} \cos(\delta_{\text{slack}}) \quad \text{(B.27b)}
\]

while the derivatives with respect to branch currents remain equal to (18). In the second case (voltage amplitude and phase measurement directly employed), the derivatives related to the slack bus can be found approximating \(\sin(\delta_{\text{slack}} - \delta_v) \simeq \delta_{\text{slack}} - \delta_v\):

\[
\frac{\partial v_h^*}{\partial \delta_v} = \cos(\delta_{\text{slack}} - \delta_v) \quad \text{(B.28a)}
\]

\[
\frac{\partial v_h^*}{\partial v_{\text{slack}}} = -v_{\text{slack}} \sin(\delta_{\text{slack}} - \delta_v) \quad \text{(B.28b)}
\]

\[
\frac{\partial v_{\text{slack}}}{\partial \delta_v} = 0 \quad \text{(B.28c)}
\]

\[
\frac{\partial v_{\text{slack}}}{\partial v_h} = 1 + \sum_{j \in \Gamma_h} z_{hj}^* v_h \cos(\alpha_{xzj} + \theta_j - \delta_{\text{slack}}) \quad \text{(B.28d)}
\]

where \(\Gamma_h\) is, as already said, the path between the measured node and the slack bus.

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REFERENCES


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