Varieties of Brouwer–Zadeh Lattices
Generated by Horizontal Sums

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Let us denote by $\text{AOL}$ the class of antiortholattices and by $\text{OL}$, $\text{OML}$, $\text{BZL}$ and $\text{PBZL}^*$ the varieties of the ortholattices, orthomodular lattices, Brouwer–Zadeh lattices (in brief, BZ–lattices) and PBZ*$–lattices, respectively; recall that $\text{PBZ}^*$–lattices are the paraorthomodular BZ–lattices in which each pair consisting of an element and its Kleene complement fulfills the Strong de Morgan condition.

For any non–trivial bounded lattice $L$, we let the horizontal sum of $L$ with the two–element chain be $L$. For any non–trivial BZ–lattices $A$ and $B$, we endow the horizontal sum of bounded lattices $A \boxplus B$ with two unary operations that restrict to the Kleene complement and the Brouwer complement of $A$ and $B$; in this way, $A \boxplus B$ becomes a BZ–lattice iff one of the summands $A$ and $B$ is an ortholattice, and a PBZ$^*$–lattice iff one of them is an orthomodular lattice and the other is a PBZ$^*$–lattice.

We study the congruences of horizontal sums of BZ–lattices.

For any classes $C$ and $D$ of BZ–lattices, we denote:

$$C \oplus D = \{D_1\} \cup \{A \oplus B : A \in C \setminus \{D_1\}, B \in D \setminus \{D_1\}\},$$

where $D_1$ is the trivial BZ–lattice.

With the usual notation for the class operators, we study the varieties:

$$\text{OML} \vee \text{HSP}(\text{AOL}), \quad \text{OL} \vee \text{HSP}(\text{AOL}),$$
$$\text{HSP}(\text{OML} \boxplus \text{AOL}), \quad \text{HSP}(\text{OL} \boxplus \text{AOL}),$$
$$\text{HSP}(\text{OML} \boxplus \text{HSP}(\text{AOL})), \quad \text{HSP}(\text{OL} \boxplus \text{HSP}(\text{AOL}))$$

from the point of view of their relative axiomatizations with respect to each other and to $\text{PBZL}^*$ and $\text{BZL}$.